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OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2005

H. DIPLOMA ARTS
MASTER OF ARTS

MATHEMATICS — [MA541]

ADVANCED ANALYSIS II

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Time allowed: **Two** hours.
Full marks for three questions.

1. (a) Show $\|x\|_{l^8} \leqq \|x\|_{l^7}$ for all $x \in l^7$.

(b) Let X and Y be normed spaces with $T : X \to Y$ a linear mapping. If T is continuous at 0 show there exists $M > 0$ with $\|Tx\| \leqq M\|X\|$ for all $x \in X$.

(c) Let X be a normed space and Y a Banach space. Show $B(X, Y)$ is a Banach space.

2. (a) Show that the dual space $(c_0)'$ is isometrically isomorphic to l^1.

(b) A linear functional is defined on the Banach space l^8 by $\phi(x) = 2x_1 - 3x_2 + 7x_4 - 5x_6$ where $x = (x_1, x_2, ...) \in l^8$. Find the norm of ϕ.

3. (a) Let X be a normed space and $x_0 \in X$ with $x_0 \neq 0$. Use the Hahn-Banach theorem to show that there exists a bounded linear functional ϕ on X with $\|\phi\| = \frac{1}{\|x_0\|}$ and $\phi(x_0) = 1$.

(b) Show c_0 (with the usual sup norm) is not a Hilbert Space.

(c) Let K be a nonempty closed convex subset of a Hilbert space H. Show for every $x \in H$ there is a unique $y \in K$ with

$$\|x - y\| = \inf_{z \in K} \|x - z\|.$$

4. (a) Suppose H_1 and H_2 are Hilbert spaces and $T : H_1 \to H_2$ a bounded linear operator. Define the Hilbert adjoint T^* of T. Show T^* exists, is unique, and is a bounded linear operator with norm $\|T^*\| = \|T\|$.

(b) Let $S : l^2 \to l^2$ be given by

$$S(x) = (0,\, x_1,\, x_2,\, \ldots) \text{ when } x = (x_1, x_2, \ldots)$$

For $n \in \{1, 2, \ldots\}$ show $S^*(e_{n+1}) = e_n$.