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*Ollscoil na hÉireann, Gaillimh*
*National University of Ireland, Galway*

# Semester II Examinations, 2004/2005

**Front Page Template**

| | |
|---|---|
| Exam Code(s) | 4BM121 |
| Exam(s) | 4th Mechanical Engineering |
| Module Code(s) | ME402 |
| Module(s) | Advanced Mechanical Analysis and Design |
| Paper No. | 1 |
| Repeat Paper | Special Paper |
| External Examiner(s) | Prof. J. Fitzpatrick |
| Internal Examiner(s) | Prof. J.F. McNamara |
| | Prof. P.E. McHugh |

**Instructions:**
Answer 4 questions.
All questions will be marked equally.
Tables 8.1, 9.4, 11.1 and 11.2, and Figure 11.6 from Burr & Cheatham are attached.

| | |
|---|---|
| Duration | 3hrs |
| No. of Answer books | 1 |

**Requirements:**

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| Handout | |
| MCQ | |
| Statistical Tables | |
| Graph Paper | |
| Log Graph Paper | |
| Other Material | X Mathematics Tables |

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| No. of Pages | 14 Including 6 pages of tables |
| Department(s) | Mechanical and Biomedical Engineering |

1 (a) Clearly state the assumptions of axi-symmetric deformations. Assuming a cylindrical polar coordinate system ($r$, $\theta$, $z$), derive expressions for the radial strain $\varepsilon_r$ and the tangential strain, $\varepsilon_t$, in terms of the displacement field. Comment on the form of the result. **(5)**

(b) For a thick-walled axi-symmetric structure in plane stress the following are the general expressions for the radial and tangential stresses ($\sigma_r$ and $\sigma_t$) as functions of radial position, $r$, in the absence of temperature variation or angular rotation, where $E$ and $\nu$ are Young's modulus and Poisson's ratio, respectively, and $C_1$ and $C_2$ are unknown constants.

$$\sigma_r = \frac{EC_1}{1-\nu} - \frac{EC_2}{(1+\nu)r^2}$$

$$\sigma_t = \frac{EC_1}{1-\nu} + \frac{EC_2}{(1+\nu)r^2}$$

Consider the case of a cylinder subject to an internal pressure $p_i$. Use the above expressions to derive explicit formulae for $\sigma_r$ and $\sigma_t$, clearly showing each step in the derivation, and sketch the variation of the stresses as functions of $r$. **(9)**

(c) A cylindrical vessel with an internal radius of 500 mm is to be designed to withstand an internal pressure of 20 MPa, where the allowable shear stress for the vessel material is 70 MPa. Determine the external radius and the maximum magnitudes of $\sigma_r$ and $\sigma_t$. **(6)**

**2 (a)** Consider a thin-walled axi-symmetric pressure vessel of thickness $t$ with internal pressure $p$. Derive the Membrane Equation for such a vessel. In your answer you should use appropriate and clearly labelled free body diagrams and you should clearly define each symbol used. **(6)**

**(b)** A paraboloid of revolution is to be used as the shape for an observation capsule on a deep sea vehicle. The capsule is to be made of glass. Taking the equation of a parabola to be $z = ar^2$, derive expressions for the stresses $\sigma_m$ and $\sigma_t$ and simplify them as much as you can. **(8)**

Consider the case where the height of the capsule is 500 mm, the radius at the base is 500 mm and the constant $a$ has a value of 0.002 (mm$^{-1}$). Determine the required thickness of the glass to withstand an external pressure of 1 MPa, where the maximum allowable compressive stress in the glass is 30 MPa. Compare this with the thickness that would be required for a hemispherical capsule, and comment on the result. **(6)**

You are given the following expressions:

$$\sigma_m = \frac{pR_t}{2t} \qquad \sigma_t = \sigma_m\left(2 - \frac{R_t}{R_m}\right)$$

$$R_t = \frac{r}{\sin[\tan^{-1}(dz/dr)]} \qquad R_m = \frac{[1+(dz/dr)^2]^{3/2}}{d^2r/dz^2}$$

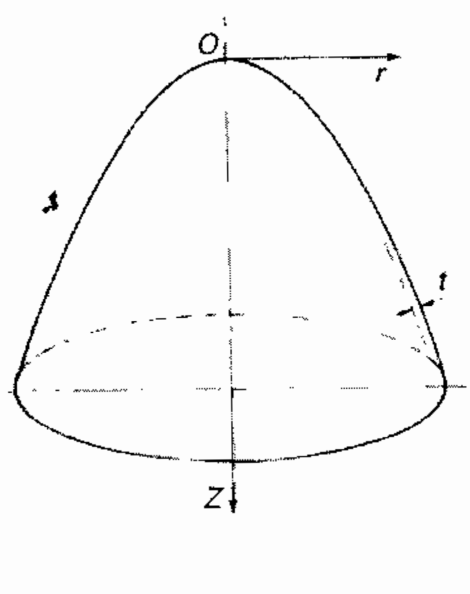


*Figure 2*

3(a) Consider an interference fit between a solid shaft of radius $n$ and a disk of outer radius $q$ made of the same metal with elastic constants $E$ and $\nu$, and density $\rho$. If the assembly is rotating with an angular speed $\omega$ derive the following relationship between the interface pressure $P_f$ and the diametrical interference $\Delta$. **(12)**

$$\frac{4P_f nq^2}{E(q^2 - n^2)} + \frac{3+\nu}{2E}\rho\omega^2 nq^2 = \Delta$$

**(b)** Consider a gear box designed for transmitting torque. A gear with a tooth root diameter of 240 mm and 20 mm thickness is assembled onto a solid shaft of 100 mm diameter with a diametrical interference of 0.12 mm. What is the fit pressure when just assembled, and when rotating at 5000 rpm? What is the maximum torque that can be transmitted by the joint assuming a friction coefficient of 0.2? You can assume $E$ = 207 GPa, $\nu$ = 0.3 and $\rho = 7.778\text{x}10^3$ kg/m$^3$. **(8)**

4 (a) Derive the following fundamental equation relating axial stress, $\sigma$, to bending moment, $M$, $2^{nd}$ moment of area, $I$, and vertical position above Neutral Axis, $y$, for a thin straight beam in bending. **(5)**

$$\sigma = \frac{-My}{I}$$

(b) Consider a uniform shaft of diameter $D$ and length $l$ shown in Figure 4. The shaft is supported by bearings at $x = 0$ and $x = l$. The shaft supports parallel forces $P$ at positions $x = l/4$ and $x = 3l/4$. There is a third bearing at the midpoint $x = l/2$ but due to machining errors this bearing is out of line by an amount $\delta$, in the opposite direction to the direction of the loads $P$. Assuming that the bearings act as simple supports, derive expressions for the bearing reaction forces and the maximum bending moment in the shaft in terms of $P$, $E$, $I$, $\delta$ and $l$. From this write down an expression for the maximum stress in the shaft. **(15)**

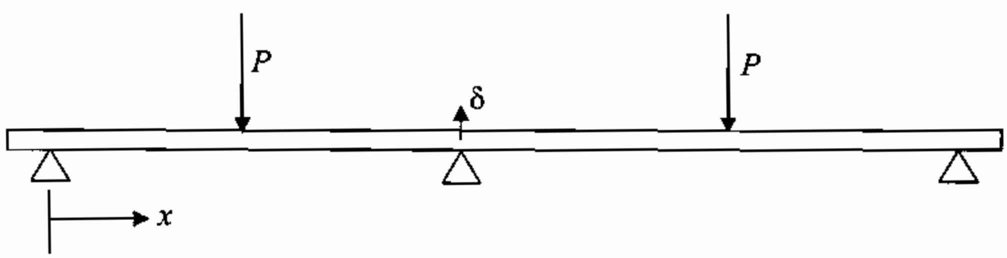


***Figure 4***

5 (a) Consider the rigid bracket attached to a wall by three rows of identical bolts, as shown in Figure 5a. There are $N_i$ bolts in each row ($i = 1,2,3$). The bracket supports a force $F$ at a distance $a$ from the wall. Derive expressions for the axial force in each bolt. **(10)**

(b) Consider the generator attached to a wall as shown in Figure 5b that transmits a maximum torque of 500 Nm. There are three rows of identical bolts supporting the generator: 3 in the top row, 2 in the middle row and 3 in the bottom row. The mass of the generator is 300 kg and its centre of gravity is 200 mm from the wall. Determine the forces in the bolts (neglecting pre-tightening) when they are supporting both the torque and the weight of the generator. **(10)**

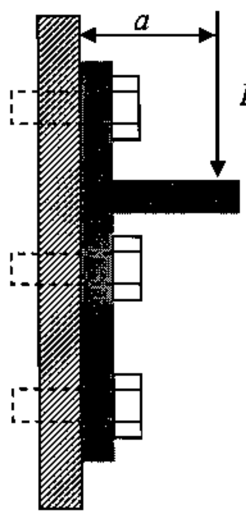


*Figure 5a*

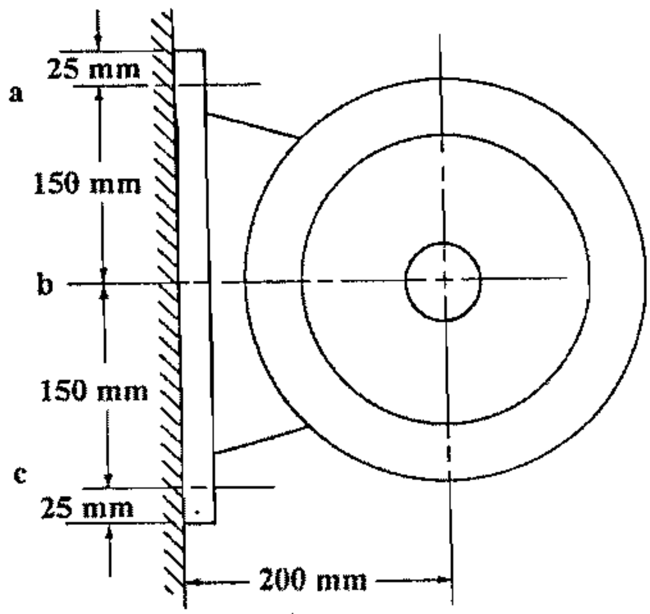


*Figure 5b*

6. The flexible support proposed for an extraterrestrial vehicle consists of four wheels and semicircular springs, readily unfolded from the body of the vehicle after landing and then clamped in the position shown in Figure 6. Determine the following in terms of $E$, $I$, vehicle weight $W$ and spring radius $r$.

(i) The downward deflection of the vehicle. **(10)**

(ii) The horizontal motion of a wheel due to the vehicle weight only. **(8)**

Comment on the relative magnitudes of the deflections and sketch the deformed shapes of the springs. **(2)**

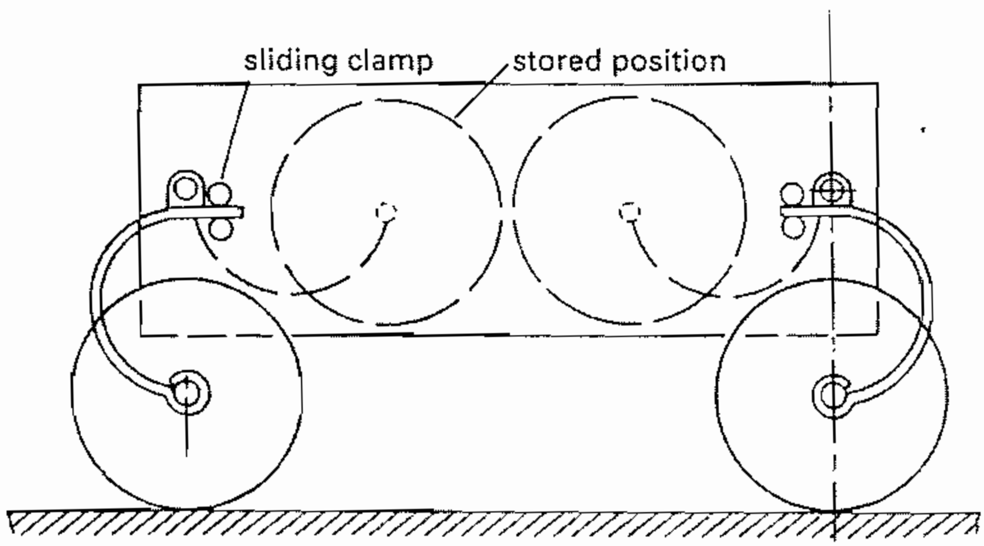


***Figure 6***

7. A pivot ball bearing is shown in Figure 7, that supports axial loads $F_a$. There are $N$ balls, each of a diameter $d$. The cone angle is $2\alpha$ and the diameter of the cone when in contact with the balls is $D$. To analyse and design such a ball bearing it is essential to be able to determine the maximum contact pressure.

(i) Write down an expression for the normal contact force at the ball-cone contact point for each ball. **(2)**

(ii) With reference to Table 11.2 and Figure 11.6 (attached) derive an expressions for the geometric parameters (B+A) and (B-A)/(B+A) for the pivot ball bearing. **(8)**

(iii) For the case of $F_a = 10$ N, $d = 3$ mm, $D = 6$ mm, $2\alpha = 50°$, $N = 9$ and all components made of steel, calculate the maximum contact pressure and determine the factor of safety based on a maximum allowable contact pressure of 4000 MPa. **(10)**

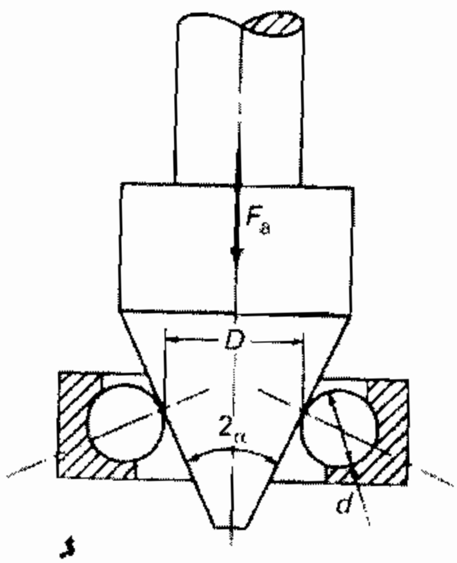


*Figure 7*