

OLLSCOIL NA hÉIREANN, GAILLIMH  
NATIONAL UNIVERSITY OF IRELAND, GALWAY

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SEMESTER 1 EXAMINATIONS, 2004-2005

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FIRST YEAR FINANCIAL MATHEMATICS AND  
SECOND YEAR ARTS

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STATISTICS [MA112, MA227]

Dr. T.C. Bailey  
Professor T.C. Hurley  
Dr. R.A. Ryan

Time allowed: *Two* hours.  
Answer three questions.

1. (a) The marks obtained by 20 students in an examination are as follows:

82, 60, 40, 41, 68, 70, 75, 78, 50, 52, 80, 41, 44, 60, 55, 46, 50, 60, 53, 52.

Construct a stem-and-leaf display for the data, and find the median  $Q_2$  and the quartiles  $Q_1$  and  $Q_3$ .

- (b) Now take the first five marks from the list above. Find the mean and the standard deviation of this *sample*.
- (c) In a Mathematics examination, the mean mark for the Algebra paper is 72, with a standard deviation of 6, while the mean mark in Calculus is 64, with a standard deviation of 3. Seán got 81 in Algebra and 71 in Algebra. Which is the better result?
- (d) An executive of 4 is to be selected from a board consisting of 7 women and 3 men. In how many ways may this be done? How many executives contain exactly 2 women?

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2. (a) On a tray there are 7 slices of chocolate cake and 5 slices of coffee cake. If a waiter randomly picks two slices of cake from the tray and serves them to customers who ordered chocolate cake, what is the probability that he has given them the correct order?
- (b) State Kolmogorov's axioms for a probability model. Use them to show that if  $A$  and  $B$  are two events in a sample space, then

$$P(A \cup B) = P(B) + P(A) - P(A \cap B).$$

- (c) The odds *in favour* of an event  $A$  are 3 to 1, and the odds *in favour* of the event  $B$  are 2 to 1. If  $A$  and  $B$  are independent, find the odds in favour of the event  $A \cup B$ .
3. (a) A company has factories in Ireland, Korea and Brazil, producing 20%, 30% and 50% respectively of its total output of a certain item. The factories in question have probabilities of 0.1, 0.3 and 0.2 of producing a faulty product. What is the probability that an item chosen at random will be faulty?
  - (b) *State* the Law of Total Probability. *State* and *prove* Bayes' Theorem.
  - (c) Assume that in the criminal court in a certain country, there is an 85% probability of a correct verdict in any given trial. Further, suppose that 99% of the defendants brought to trial are in fact guilty. Given that a defendant is found guilty by the court, what is the probability that he is in fact guilty of the crime?
4. (a) An unbalanced coin has probability  $3/4$  of turning up heads when tossed. The coin is tossed 192 times. Use the Normal approximation to the Binomial Distribution to estimate the probability that we get exactly 64 tails.
  - (b) The actual amount of milk in a one-litre carton has a Normal distribution with a standard deviation of 4 millilitres. What should the mean fill of the cartons be if at most 1% of them are to contain less than one litre? (Note: there are 1,000 millilitres in one litre.)
  - (c) *State* the Central Limit Theorem. The distribution of weights of all NUI Galway students has a mean of 164 lbs and a standard deviation of 15 lbs. Use the Central Limit Theorem to estimate the probability that the combined weight of a sample of 25 students exceeds 4225 lbs.