2066

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS 2005-2006

CS 450 COMMUNICATIONS AND INFORMATION THEORY

Dr. D. Johnson
Dr. D. Fay

Time Allowed: Two hours
Answer 3 of 4 questions. All questions carry equal marks.

Question 1.

(a) Explain the concept of a typical sequence.

(b) Using the concept of typical sequences give an intuitive proof of Shannon's ***Source*** coding theorem. That is, show that the maximum possible compression possible is $2^{nH(X)}$, where n is the block length, H denotes the entropy function and X is the source.

(c) Given a binary source X, with alphabet, $A_X = \{0,1\}$ and associated probabilities, $P_X=\{0.5, 0.5\}$, explain using Shannon's source coding theorem why no *lossless* compression is possible?

(d) An Alphabet $A= \{a_1,a_2,\ldots,a_N\}$ with associated probabilities $P = \{p_1,p_2,\ldots,p_N\}$ is encoded using a decodable code. Given that the cost of a codeword c_i is defined as:

$$c_i = 2^{-l_i}$$

where l_i is the length of the codeword. Prove that:

$$\sum_i c_i \leq 1$$

Exam continued on next page

Question 2.

(a) Given the probabilities in Table 1 below construct an arithmetic code for the following piece of text: 'tory'.

Table 1.

Character	Probability
A	0.1
B	0.1
C	0.1
I	0.1
L	0.1
O	0.1
R	0.1
T	0.1
P	0.1
U	0.05
Y	0.05

(b) How many bits are required to transmit this message?

(c) Given that:

$$I(X;Y) = H(X) - H(X \setminus Y)$$

prove:

$$I(X;Y) = H(Y) - H(Y \setminus X)$$

where the above are defined in the usual way.

(d) Consider a binary ***erasure*** channel. Let the probability of sending a 1 and receiving a 1 be 1-f_1 and the probability of sending a 0 and receiving a 0 be 1-f_2. The source X, produces bits with probability: $P_x = \{p_0, p_1\}$. Calculate in terms of p_0, f_1 and f_2 :

i. The entropy of Y,

ii. Given $p_0 = 0.3$, $f_1 = 0$ and $f_2 = 0.3$ calculate

a) The conditional entropy of Y given X,

b) The mutual information between X and Y.

Question 3.

(a) Prove the following Fourier transform properties:

i. $x(t) * y(t) \Leftrightarrow X(f) \times Y(f)$

ii. $x(t)\cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$

(b) Prove that a signal $x(t)$ with $X(f) = 0$ for $f > f_{\max}$ sampled at f_s, where is $f_s \geq 2\, f_{\max}$ can be reconstructed exactly using:

$$x(t) = \sum_{n=-\infty}^{\infty} 2f'/f_s x(n/f_s)\operatorname{sinc}(2f'(t - n/f_s))$$

where $f_{\max} \leq f' \leq f_s - f_{\max}$ is an arbitrary frequency.

(c) A signal is bandlimited to 54KHz. What sampling rate do we require for a guard band of 15KHz ?

Question 4.

(a) Given a deterministic input, sketch (no derivations necessary) and compare the spectrums of:

i. Conventional AM,
ii. SSB-AM,
iii. FM, and
iv. PM.

(b) Show that the spectrum of a DSB-SC AM signal when the message signal is ***random*** can be expressed as:

$$\frac{1}{4}A_c^2\left(S_m(f - f_c) + S_m(f + f_c)\right)$$

where above defined in the usual way.

(c) Briefly explain Frequency Division Multiplexing.