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OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

*SEMESTER I EXAMINATIONS 2005/2006*

M.Sc. Degree Examination

COMMUNICATIONS SYSTEMS THEORY
**CS457 — Digital Image Processing and Analysis**

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Time allowed: **Two** hours.
Attempt **THREE** questions.

Q1. (a) Given a set of distinct points $\omega^n = \{x_0, x_1, \cdots, x_n\}$ and a real-valued function $f(x)$, show that there is a unique polynomial of degree $n$ that interpolates $f(x)$ at those points.

(b) Suppose that the interpolation points are equally spaced (i.e., $h = (x_n - x_0)/n$ and $x_k = x_0 + kh$). Define $\mathcal{L}(x)$, the piecewise linear interpolant to $f(x)$ on $\omega^n$ and give a formula for it. Show that

$$\max_{x_0 \le x \le x_n} |f(x) - \mathcal{L}(x)| \le \frac{h^2}{8} \max_{x_0 \le x \le x_n} |f''(x)|$$

(c) Describe the method of *bilinear interpolation* for increasing the size of an image.

Q2. (a) Show that the Laplacian Operator

$$\Delta^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

is *isotropic* (invariant under rotation).

(b) Show how to derive a discrete Laplacian from a truncated Taylor Series. In what sense is this operator isotropic?

(c) Describe algorithms for

i. Edge Detection,

ii. Image Sharpening,

based on the Discrete Laplacian Operator.

Q3. (a) The Discrete Fourier Transform of a signal (vector) of length $M = 2^n$ is

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) \exp(-2\pi i u x / M) \qquad \text{for } u = 0, 1, \ldots, M.$$

Show how to construct the Fast Fourier Transform (FFT) Algorithm.

(b) Show that computing the FFT of a signal of length $M = 2^n$ requires $\frac{1}{2} M \log_2 M$ complex multiplications, and $M \log_2 M$ complex additions.

(c) Describe the Gaussian Lowpass Filter and its application for image enhancement in the frequency domain.

Q4. Write a short note on **three of the following topics**

(a) Histograms and Histogram equalisation for image processing.

(b) The use of the $AND$, $OR$ and $NOT$ logic operators for image processing.

(c) Order statistics for noise reduction.

(d) Homomorphic processing in the frequency domain.