

Ollscoil na hÉireann, Gaillimh
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Exam Code(s) 2BA5
Exam(s) 2nd Arts (Economic and Social Studies)
St. Angela's College, Sligo

Module Code(s) EC222
Module(s) Techniques of Analysis in the Social Sciences

Paper No.
Repeat Paper No

External Examiner(s) Professor Vincent Munley
Internal Examiner(s) Brendan Kennelly
Stephen McNena

Instructions: There are two sections in this exam: A and B. Please note the instructions for each section.

Please use separate Answer Books for sections A and B.

Duration 2 hours
No. of Pages 6
Department(s) Economics
Course Co-ordinator(s) Stephen McNena, St. Angela's College, Sligo

Requirements:

MCQ
Handout
Statistical Tables
Graph Paper
Log Graph Paper
Other Material

SECTION A STATISTICS

Answer any 6 of the 8 questions. 10 minutes per question.

1. The managing director of a chain of 25 camera shops gathered the following data about the monthly sales of a particular type of camera across each of the chain's 25 outlets:

65	98	55	62	79	59	51	90	60	61	75	72	56
70	62	66	80	94	79	63	73	65	61	71	85	

The data is to be organised into a frequency distribution.

- (a) How many classes should be used?
 - (b) What class width should be used?
 - (c) Organise the data into a frequency distribution.
 - (d) Comment on the shape of the frequency distribution.
 - (e) Determine the relative frequency distribution.
 - (f) Construct a histogram.
2. (a) In June you bought 300 concrete blocks at €2 per block. In August you bought an additional 400 blocks at €1.50 each. In November the price had dropped to €1.40 when you bought an additional 500 blocks. What is the weighted mean price per concrete block?
- (b) You invest some of your savings in a managed fund. The annual percentage returns earned by the fund for the last 5 years are outlined in the table below. Calculate the geometric mean annual return earned by the fund.

9.4%	13.8%	11.7%	11.9%	4.7%
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- (c) (i) Calculate the mean, median and mode of the following set of numbers.
- 8, 8, 9, 6, 9, 7, 9, 3, 21
- (ii) Explain why the median is sometimes a preferable measure of central location than the mean.

3. (a) A student is taking two courses: physics and botany. The probability the student will pass physics is 0.80, and the probability of passing the botany course is 0.85. The probability of passing both subjects is 0.70. What is the probability of passing at least one subject?
- (b) A small factory has two machines that sometimes break down. The probability of the first machine being available is 0.80 and the probability of the second machine being available is 0.75. What is the probability that neither machine breaks down? What is the probability that both machines break down?
- (c) Sarah is the owner of the Cohen cafe. Sarah offers free refills on all coffee orders. She gathered the following information on coffee refills.

X = no. of refills	P(x)
0	0.30
1	0.40
2	0.20
3	0.10

- (i) Compute the mean and standard deviation for the distribution of number of refills.
- (ii) If a customer buys a cup of coffee, what is the chance that they have at least 2 refills?
4. (a) The industry norm suggest that 5% of new buses need repairs within the first year. Last year your firm sold 50 new buses.
- (i) What is the probability that none of the buses will require a repair?
- (ii) What is the probability that exactly one of the buses will require a repair?
- (iii) Compute the mean and standard deviation of this probability distribution.
- (b) The mean starting salary for college graduates in 2004 was €21,000. Assume that the distribution of starting salaries follows the normal distribution with a standard deviation of €3,300. What percent of the graduates have starting salaries:
- (i) Between €19,000 and €23,000?
- (ii) More than €30,000?
- (iii) Between €23,000 and €30,000?
5. (a) School records show that the results of the entrance exam are normal distributed with a mean score of 60 and a standard deviation of 12. You select a random sample of 9 students from the school. Compute the probability that the sample mean is:
- (i) greater than 63.
- (ii) less than 56
- (iii) between 56 and 63.
- (b) The mean rent for a one-bedroom apartment in California is \$1,200 per month. The distribution of the monthly rents does not follow the normal distribution. In fact, it is positively skewed. What is the probability of selecting a sample of 50 one-bedroom apartments and finding the mean to be at least \$1400 per month? The standard deviation of the sample is \$250.

6. The manager of a branch of a nationwide chain of music stores wants to study certain characteristics of her customers. Specifically, how much the customers spend, and whether they own a hi-fi or not. The results from a sample of 70 customers are as follows:

$$\bar{x} = \text{€}21.34 \quad s = \text{€}9.22$$

57 customers own a hi-fi

- (a) Set up a 95% confidence interval for the population mean amount spent in the shop.
(b) Set up a 99% confidence interval for the proportion of customers who own a hi-fi.
7. (a) The Sligo Chamber of Commerce wants to estimate the average time workers spend travelling to work. A sample of 15 workers reveals the following number of minutes spent commuting:

29	38	38	33	38	21	45	34
40	37	37	42	30	29	35	

The standard deviation of the sample data is 6 minutes. Develop a 98% confidence interval for the population mean.

- (b) Policymakers in the Government want an estimate of the proportion of the population that support higher taxes. The estimate is required to be within 0.04 of the true proportion. Assume a 95% level of confidence. How large a sample is required?
8. The manufacturer of the Sparky bulb claims the mean life of the bulb is 5,000 hours. The standard deviation is 500 hours. You buy 48 bulbs and find that the mean life achieved in your offices is 4,000 hours.

Is your experience different from that claimed by the manufacturer at the 95% confidence level? You must follow all six steps in your answer.

SECTION B MATHS

Answer any **6 of the 8** questions. 10 minutes per question.

1. Consider the demand function represented by the equation:

$$Q_d = 100 - 5P$$

- (a) Express Total Revenue (TR) as a function of Q_d .
- (b) Determine the equation for Marginal Revenue.
- (c) Using differentiation, calculate the quantity where TR is maximised.
- (d) Calculate the price where TR is maximised.
- (e) Calculate the maximum Total Revenue.

2. (a) Consider the following Cobb-Douglas production function:

$$Q = f(K, L) = AK^aL^b$$

where Q is output, K is capital, and L is labour. Transform this equation into a linear model using logarithms.

- (b) A firm that makes steel components sells their product for €8. The cost function is represented by the equation:

$$TC = 2000 + 4Q$$

- (i) Calculate the breakeven quantity.
 - (ii) Determine the Total Revenue, Fixed Cost, Variable Cost and Total Costs.
3. Consider an open economy described by the following equations:

$$C = 300 + 0.8Y_d$$

$$I (\text{autonomous}) = 400$$

The Government's spending is 600, financed by a 20% tax rate on all income. Exports are 800, and imports are 30% of disposable income.

Note that C , Y and M are endogenous, and I , G , t and X are exogenous constants.

- (a) Solve for the equilibrium national income, Y .
- (b) Then calculate Y_d , C and M .
- (c) Determine the Marginal Propensity to Consume and the Marginal Propensity to Save.
- (d) The government decide to increase their expenditures to 650. Calculate the new value for National Income (Y).

4. Using **matrix inversion**, solve the following systems of equations for Q_d , Q_s , and P . Also calculate Total Revenues. Show all calculations.

$$\begin{aligned}Q_d &= 500 - 9P \\ Q_s &= -100 + 6P\end{aligned}$$

5. Using **matrix inversion**, solve the following systems of equations for Y and C . Also calculate consumption and savings. Show all calculations.

$$\begin{aligned}Y &= C + I \\ C &= 500 + 0.75 Y \\ I &= 600\end{aligned}$$

6. Assume a Total Cost function of the form:

$$TC = 400 - 3Q + 0.25Q^2$$

- (a) Derive a function for Average Cost (AC) and Average Fixed Cost (AFC).
- (b) Derive a function for Marginal Cost (MC).
- (c) Find the levels of output that minimise AC and MC.
- (d) Calculate the minimum AC.
- (e) Using algebra, confirm that $AC = MC$ when AC is at a minimum.

7. A firm faces the following demand and Total Cost functions:

$$\begin{aligned}P &= 125 - Q \\ TC &= 500 + 5Q + 0.5Q^2\end{aligned}$$

where Q is measured in hundreds of tons.

- (a) Determine the equations for Total Revenue, Marginal Revenue, Average Cost, Marginal Cost and Profit.
 - (b) Derive the profit-maximising level of output
 - (c) Calculate the maximum profit.
 - (d) Derive the price and Marginal Revenue at this level of output.
8. A person with a weekly income of €100 receives utility (U) from consuming two products (A , B), with prices of $A = €2$ and $B = €10$. They face a utility function as follows:

$$U = 5A^{0.3}B^{0.2}$$

Determine the utility-maximising combination of the two goods.