OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 1999

B.E. DEGREE EXAMINATION

STRUCTURAL ANALYSIS

First Paper

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Time allowed : ***Three*** hours

Answer ***four*** questions in all -
Two from Section A, and ***two*** from Section B

Please use a separate answer book for each section

NOTES
The use of electronic calculators is allowed;
All dimensions are in **mm**, unless noted otherwise;

For Section B, some expressions relating to Question 6 appear at the end of the paper.

SECTION A

Use a separate answer book for each section

Answer *two* questions from Section A

Question 1

(a) Figure Q1 shows a three-span continuous simply-supported steel beam, subject to the factored loading shown. It is also fully restrained laterally. Calculate and draw the elastic bending moment diagram (BMD). Assuming a uniform **compact** cross-section throughout, chose a suitable Universal Beam section.

(b) Assuming welded splices at supports B and C, choose three suitable **plastic** Universal Beam sections on the basis of a non-uniform cross-section design. Draw the collapse BMD.

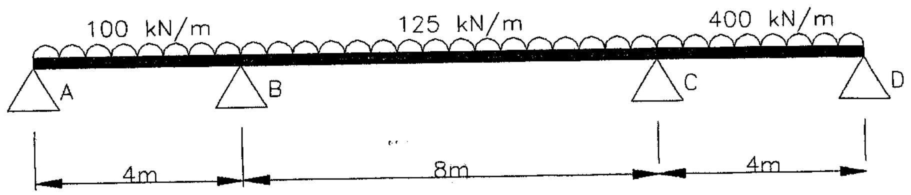


Figure Q1

Question 2

(a) State the Static and Kinematic Theorems of plastic collapse and from them derive the Uniqueness Theorem of plastic collapse.

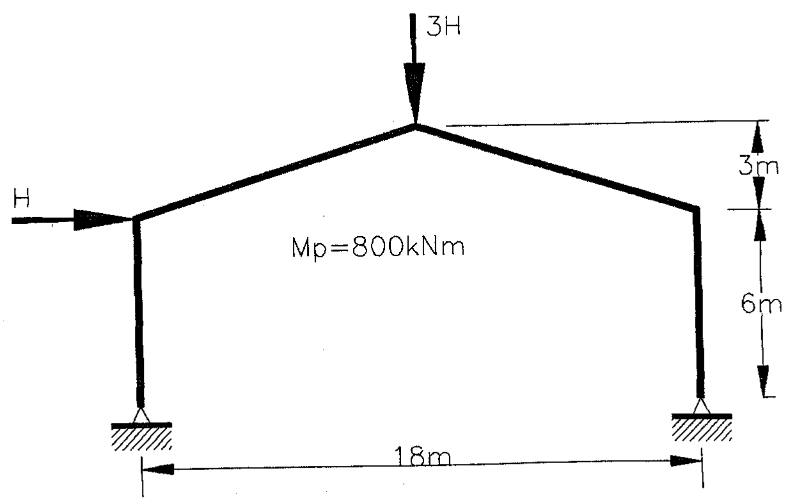
(b) Explain how these theorems are utilised in practice such as in the case of a complex framework in which many plastic collapse mechanisms are possible.

(c) Use these theorems to derive an expression for the minimum required plastic moment of resistance $\mathbf{M_{pl}}$ for a propped cantilever of span $\mathbf{L}$, loaded by a uniformly distributed loading, $\boldsymbol{w}$.

Question 3

(a) Figure Q3 shows a pitched portal frame, which is of uniform cross-section throughout with **Mpl=800kNm** and has pinjointed feet. Find the value of **H**, which causes ultimate plastic collapse of the frame.

(b) Calculate and draw the corresponding ultimate plastic collapse bending moment diagram.



FigureQ3

Section B

4. A post-tensioned beam is required to span 15m. At transfer, it will carry its self-weight, G_{k1}, whilst in service it will carry additional dead loading, G_{k2}, and live loading Q_k, as given below.

If the minimum required prestressing force at midspan is 2700 kN:

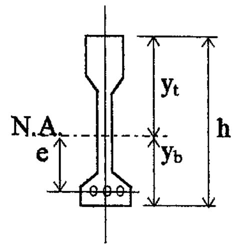
(i) determine a suitable eccentricity for the centroid of the tendon group at midspan and at support;
(ii) calculate top and bottom fibre stresses at transfer and in service;
(iii) describe briefly the various factors which give rise to losses in prestress, and calculate those due to friction and elastic shortening.

Section properties

Concrete:
$A_c = 3.23 \times 10^5$ mm^2
$I_c = 4.97 \times 10^{10}$ mm^4
y_t = 614 mm
h = 1200 mm

Material properties

Concrete:
E_{cu} = 31 kN/mm^2
E_{ci} = 29 kN/mm^2
f_{cu} = 50 N/mm^2
f_{c14} = 40 N/mm^2
(transfer at 14 days)



Loading
G_{k1} = 12 kN/m
G_{k2} = 20 kN/m
Q_k = 50 kN/m

Steel:
E_s = 200 kN/mm^2
f_{pu} = 1760 N/mm^2

Allowable stresses in concrete

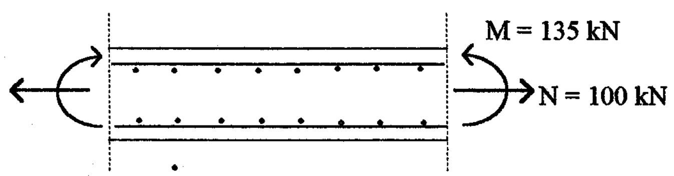
at transfer:
$f_{ci} = 0.5\, f_{c14}$
f_{ti} = -2.3 N/mm^2

in service:
$f_{cs} = 0.45\, f_{cu}$
f_{ts} = -2.6 N/mm^2

Kern limits $e_{min} = -Z_b/A + (M_s+Z_b f_{ts}) / \alpha.P$ $\qquad e_{max} = Z_t/A + (M_i-Z_t f_{ti}) / P$

Fig. Q4 Post-tensioned beam data

5. The section of reservoir wall shown in Fig. Q5 is subjected, under service conditions, to a horizontal bending moment of 135 kNm combined with a direct tensile force of 100 kN, for a metre height of wall. The wall is 500 thick and is reinforced horizontally with T16 bars at 125 centres in each face. Calculate the design surface crack width in accordance with BS8007.



f_{cu} = 35 N/mm^2

f_y = 35 N/mm^2

cover = 40mm

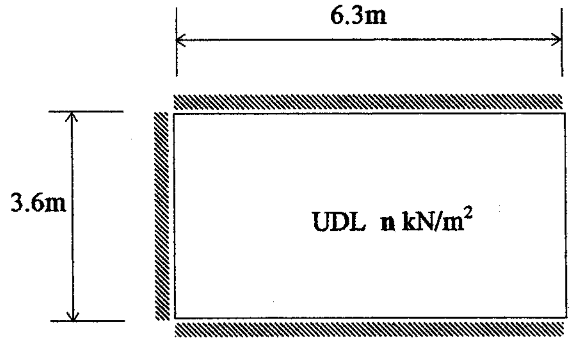
Fig. Q5 Plan

6. (a) Discuss *briefly* the essential differences between the Hillerborg and yield-line methods of slab analysis.

(b) The rectangular slab shown in fig. Q6 is fully fixed on three sides and free along one of the shorter sides. It supports a uniformly distributed load, **n** kN/m^2, and is isotropically reinforced, with ultimate moments of resistance as given below.

(i) Sketch yield-lines for two possible failure modes;

(ii) Use either of your assumed modes of failure to calculate load **n** (kN/m^2) at failure.



Ultimate moments of resistance (per metre length of slab)

Sagging:
m = 8 kNm

Hogging:
m' = 12 kNm

Fig. Q6

Expressions relating to Question 5:

Reinforced concrete sections in bending and direct tension

(i) with bending predominant:

$$\left(\frac{100A_s}{bd}\right)_{effective} = \left(\frac{100A_s}{bd}\right)\left(\frac{e+\frac{h}{2}-d}{e+\frac{h}{2}-\frac{x}{3}}\right)$$

with:

A_s	area of steel adjacent to tensile face
b	breadth of section
h	overall depth of section
d	depth to centroid of steel area A_s
x	depth to neutral axis
e	is the eccentricity M/N where N is the axial force and M is the initial bending moment (referred to the centroidal axis)

(ii) with direct tension predominant:

$$f_{s1} = \frac{1}{2A_{s1}}\left(N + \frac{M}{d-\frac{h}{2}}\right) \quad \text{and} \quad f_{s2} = \frac{1}{2A_{s2}}\left(N - \frac{M}{d-\frac{h}{2}}\right)$$

additional terms in the above are:

A_{s1}	area of steel at face which is *most* heavily stressed in tension
f_{s1}	stress in steel area A_{s1}
d	depth to centroid of steel area A_{s1}
A_{s2}	area of steel at face which is *least* heavily stressed in tension
f_{s2}	stress in steel area A_{s2}
d'	depth to centroid of steel area A_{s2} (take d'= h-d)