GX511

OLLSCOIL na hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 1999

B.E. DEGREE EXAMINATION

COMPUTATIONAL METHODS IN CIVIL ENGINEERING

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Time allowed: *Three* hours
Answer *five* questions

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1. The deflection w of a thin plate of flexural rigidity D when subjected to a lateral loading q is given by the biharmonic equation

$$\nabla^4 w = \frac{q}{D}$$

A square plate of side l has a hole in the centre as shown in Figure 1. The plate is simply supported along the four edges and is also simply supported along the edges of the hole. Use the finite difference method to find the deflection and bending moments at the internal grid points of the plate when it is subjected to a uniform transverse load of q_0N/m^2. Use a grid spacing of l/5. Assume $\nu = 0.3$.

Note:

$$M_{xy} = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$

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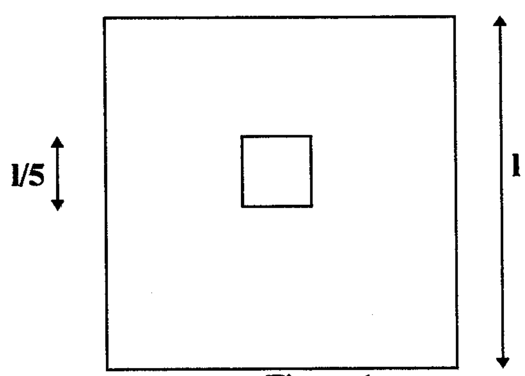

Figure 1

2. The potential energy of an axially loaded structural element may be written as

$$I = \frac{1}{2}\int_L EA\left(\frac{du}{dx}\right)^2 dx - \int_L p(x)u\,dx$$

where u(x) is the axial displacement, p(x) is the axial loading, L is the member length, A is the cross-sectional area of the member and E is Young's Modulus for the material.

In a finite element discretisation of a pin-jointed structure, quadratic truss elements are used. Using a variational approach, derive the element stiffness matrix and equivalent nodal load vector. Evaluate any two terms of the stiffness matrix and of the nodal load vector when the element is subjected to a uniformly distributed axial load p_0 per unit length.

3. The seepage of groundwater under dams may be described by the seepage equation

$$D_x \frac{\partial^2 \phi}{\partial x^2} + D_y \frac{\partial^2 \phi}{\partial y^2} = 0$$

where D_x and D_y are the coefficients of permeability and ϕ is the piezometric head measured from the bottom of a confined aquifer. The boundary conditions consist of known values of ϕ beneath the water and a zero seepage condition ($S = -(D_x \frac{\partial \phi}{\partial x} n_x + D_y \frac{\partial \phi}{\partial y} n_y) = 0$) on the other boundaries.

Use Galerkin's weighted residual method to derive the finite element equations for this problem.

4. For a curved two-dimensional isoparametric finite element, the mapping between the local ξ,η co-ordinate system and the global x,y system is defined by the relations

$$x = \sum_{i=1}^{n} N_i(\xi,\eta)\, x_i \quad ; \quad y = \sum_{i=1}^{n} N_i(\xi,\eta)\, y_i$$

When the element is used for C^0 problems, the element equations contain integrals of the form

$$\int_{A^{(e)}} f\left(\phi, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}\right) dx\,dy$$

where $\phi = \phi(\psi,\eta)$. Carry out the transformations necessary to express the integral expression in terms of the local co-ordinate system.

5. A longitudinal section through an 8m wide concrete arch bridge is shown in Figure 2. The bridge is subjected to a knife edge loading of 120kN/m width. Describe the procedure which you would follow to carry out a finite element analysis of this structure using the ANSYS package. You should include, inter alia, the following in your discussion : - solid modelling, element selection and properties, mesh generation, application of loads and boundary conditions, solution procedure, post-processing and error checking.

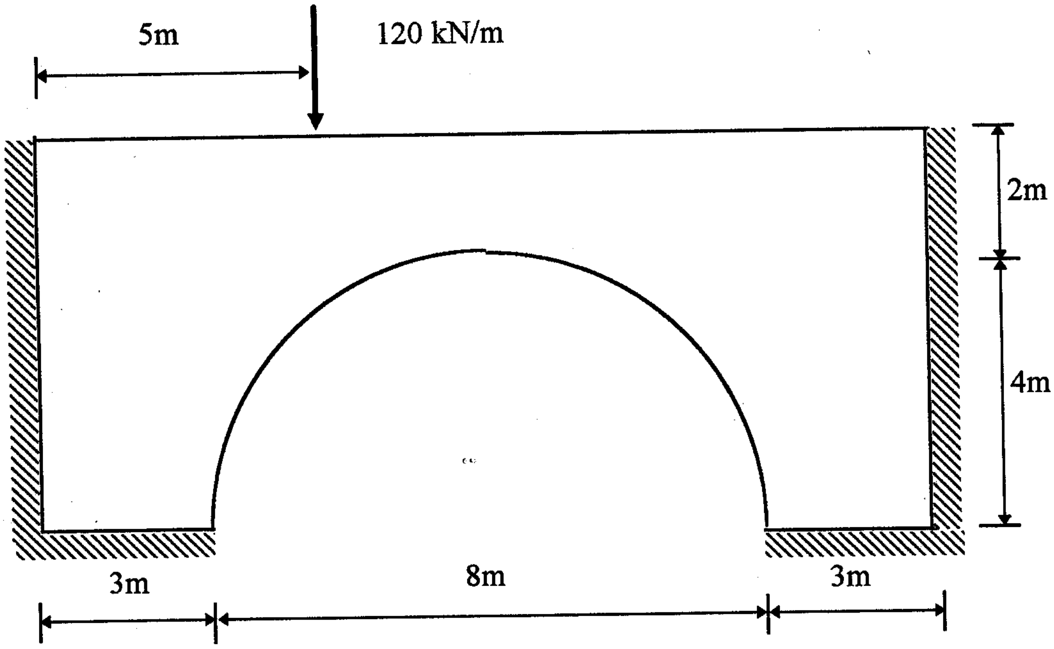

Figure 2

6. The St. Venant torsion of a prismatic shaft made from a single isotropic elastic material is described by the differential equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + 2G\theta = 0$$

where G is the shear modulus of the material, θ is the angle of twist per unit length of shaft and ϕ is Prandtl's torsion function, which is zero on the boundary. The shear stress components are given by

$$\tau_{zx} = \frac{\partial \phi}{\partial y} \quad ; \quad \tau_{xy} = -\frac{\partial \phi}{\partial x}$$

Derive a set of boundary element equations for this problem using constant elements. Show how the shear stresses at internal points are calculated. Discuss the formulation of a boundary element model for the torsion of a shaft of elliptical cross-section.

7. In the context of the computer implementation of the Finite Element Method, discuss four of the following :

(i) Co-ordinate Transformation
(ii) Assembly of System Equations
(iii) Introduction of Boundary Conditions
(iv) Equation Solution Schemes
(v) Numerical Integration Schemes