GX546

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SPRING EXAMINATIONS 1999

THIRD YEAR ELECTRONIC ENGINEERING EXAMINATION

SIGNALS AND COMMUNICATIONS

Professor L. E. Davis
Professor D.J. Wilcox
Dr. E. Jones

Duration of Examination: **3 hours**

Instructions to candidates: Answer **FIVE** questions.
All questions carry equal marks.

1. (a) The exponential Fourier series of a periodic signal *x(t)* is given by:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

[8 marks]

Show that the coefficients c_k may be computed as:

$$c_k = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-jk\omega_0 t} dt$$

(b) Obtain the trigonometric Fourier series representation of the waveform in Fig. 1.
Note that:

$$\int x \sin(ax) dx = \frac{1}{a^2}\left[\sin(ax) - ax\cos(ax)\right].$$

Sketch the spectrum of the signal.

[12 marks]

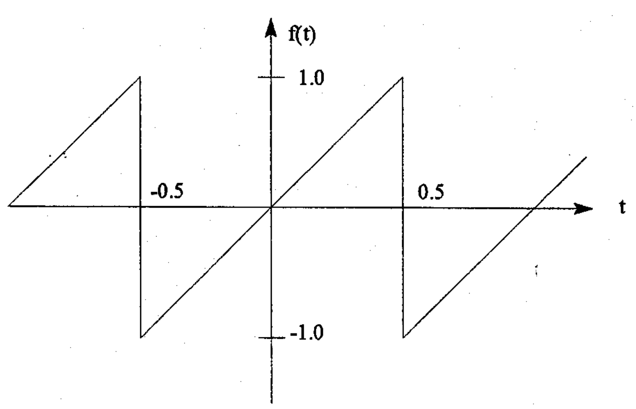

Fig. 1

[cont'd...]

2. (a) Prove the following properties of the Fourier Transform:

(i) $\mathcal{F}[f_1(t)*f_2(t)] = F_1(\omega).F_2(\omega)$, where '*' denotes convolution;
(ii) $\mathcal{F}[g(t)\, e^{j\omega_0 t}] = G(\omega-\omega_0)$;
(iii) $\mathcal{F}\left[\frac{d}{dt}f(t)\right] = j\omega F(\omega)$.

[6 marks]

(b) Find the Fourier transforms of the following functions:

(i) the single-sided exponential $f(t)=\exp(-at)u(t)$, $a>0$
(ii) the two-sided exponential $f(t)=\exp(-a|t|)$, $a>0$

[7 marks]

(c) Find the Fourier transform of the function:

$$f(t) = A\text{rect}\frac{t}{\tau}\cos(\omega_0 t)$$

i.e. a "gated" cosine pulse of duration τ and amplitude A (hint: use the frequency shifting property of the Fourier transform). Sketch the resulting spectrum.

[7 marks]

3. (a) Given the following expression for the squared magnitude frequency response of a Butterworth filter, as well as a pair of frequencies and corresponding gain values (in dB), derive an expression for the order of a Butterworth filter, in terms of these frequencies and gain values.

$$|H(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$$

[7 marks]

(b) Calculate the order of the Butterworth low-pass filter which has the following specifications on its magnitude response:

$$|H(j\omega)| = -1.8\text{ dB at } \omega_1 = 1500\text{ rads/s}$$
$$|H(j\omega)| = -24\text{ dB at } \omega_2 = 2500\text{ rads/s.}$$

[5 marks]

(c) Calculate the order of the Chebyshev Type I low-pass filter which satisfies the following requirements:

$$1.0 \geq |H(j\omega)| \geq 0.95,\ f \leq 7.5\text{ kHz}$$
$$|H(j\omega)| \leq 0.05,\ f \geq 12\text{ kHz.}$$

[8 marks]

4. (a) Give the transfer function for the Sallen-Key second-order low-pass active filter. If such a filter is designed with f_o = 10 kHz, ζ = 0.25 and R = 2kΩ, calculate a suitable value for C. What is the value of k ? Compute the gain in dB of the filter at f_o.

[7 marks]

[cont'd…]

(b) When a white noise signal is applied to a certain filter, the autocorrelation function of the filter output is found to be:

$$R_{yy}(\tau) = \frac{1}{2\tau} e^{-\left|\frac{t}{\tau}\right|}$$

Show that the corresponding power spectral density is:

$$S_y(\omega) = \frac{1}{1 + \omega^2 \tau^2}$$

[7 marks]

(c) Describe the following physical sources of noise (i) thermal noise, and (ii) shot noise. In both cases, give an approximate expression for the power spectral density.

[6 marks]

5. (a) An AM (DSB-LC) broadcast station transmits and average carrier power output of 40 kW, and uses a modulation index of 0.707 for a sinusoidal modulating signal. Calculate:

(i) the total average power output
(ii) the transmission efficiency
(iii) the peak amplitude of the output if the antenna is represented by a 50Ω resistive load.

[6 marks]

(b) Using the Fourier series, describe the principles of operation of the chopper modulator. Describe the process when (i) a unipolar (i.e. amplitude 0 to +1) rectangular pulse is used, and (ii) a bipolar (i.e. amplitude –1 to +1) rectangular pulse is used. In both cases, you may assume an even-symmetric rectangular pulse.

[7 marks]

(c) A sinusoidal modulating signal $f(t)=\cos(2000\pi t)$ is input to a chopper modulator where it is multiplied by a bipolar rectangular pulse of period T=100 μsec. The output of the multiplier is passed through a low-pass filter.

(i) Determine the minimum and maximum bandwidth of the low-pass filter if the output is to be a DSB-SC waveform corresponding to $f(t)$ (hint: at what frequency does the second significant harmonic of the square wave occur?).
(ii) Determine an expression for the output of the multiplier and low-pass filter.

[7 marks]

6. (a) For a sinusoidal modulating signal, derive an expression for the Fourier series of the corresponding frequency-modulated signal (assuming wideband FM).

[6 marks]

(b) A 1-GHz carrier is frequency modulated by a 10 kHz sinusoid so that the peak frequency deviation is 100 Hz. Determine:

(i) the approximate bandwidth of the FM signal
(ii) the bandwidth if the modulating signal amplitude were doubled
(iii) the bandwidth if the modulating signal frequency were doubled
(iv) the bandwidth if both the modulating signal amplitude and frequency were doubled.

[6 marks]

[cont'd...]

(c) The angle-modulated waveform $v(t)=A\cos(\omega_c t + \beta\sin\omega_m t)$ is applied to the input of an RC high-pass filter with the following transfer function:

$$H(s) = \frac{sRC}{1 + sRC}$$

If $\omega << (1/RC)$ in the frequency band of interest, show that the output signal is both amplitude modulated and frequency modulated. Obtain an expression for the modulation index.

[8 marks]

7. (a) Explain why it is necessary to low-pass filter an analogue signal before it is sampled and converted into digital form.

[4 marks]

(b) Find the transfer function, $H(z)$, and hence the frequency response $H(e^{j\theta})$ of the system described by the following difference equation:

$$y(n) = b_0 x(n) + b_1 x(n-1) - a_1 y(n-1)$$

[8 marks]

(c) From the expression for the frequency response, obtain an expression for the magnitude response at DC and at half the sampling frequency.

[8 marks]