OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

B.E. DEGREE IN ELECTRONIC ENGINEERING

DIGITAL SIGNAL PROCESSING

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Duration of Examination : ***Three*** hours

Instructions: Answer **five** questions.
All questions carry equal marks.

1. A digital sinusoidal oscillator is shown in Figure 1.

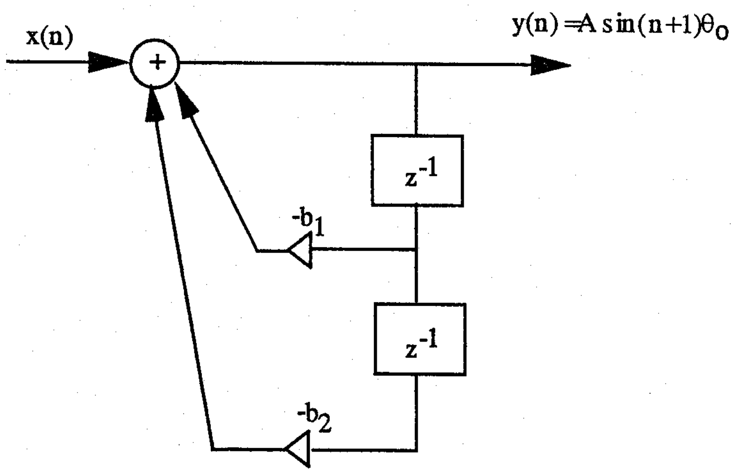

Figure 1

(a) Assuming θ_0 is the resonant frequency of the digital oscillator, find the values of b_1 and b_2 for sustaining the oscillation.

(b) Write the difference equation for Figure 1.Assuming $x(n) = (A\sin\theta_0)\delta(n)$, and $y(-1) = y(-2) = 0$, show, by analysing the difference equation, that the application of an impulse at n=0 serves the purpose of beginning the sinusoidal oscillation, and prove that the oscillation is self-sustaining thereafter.

[cont'd...]

(c) By setting the input to zero, and under certain initial conditions, sinusoidal oscillation can be obtained using the structure in Figure 1. Find these initial conditions.

(d) Using the values obtained for b_1 and b_2 in (a) above, show that

$$\Delta f_o = \frac{f_s \, \Delta b_1}{4\pi \sin\left(\frac{2\pi f_o}{f_s}\right)}$$

where fs = sampling frequency (16 kHz) and = desired frequency of oscillation. Show that the highest frequency resolution (Δf_o) that is obtainable for this oscillator is 0.078 Hz. You may assume that b_1 is represented by a K-bit number (fractional arithmetic) and Δb_1 is given by

$$\Delta b_1 = \frac{1}{2^{K-2}} \text{ where K = 16.}$$

2. Consider the following analogue system with a transfer function H(s):

$$H(s) = \frac{\alpha}{s+\alpha} \Rightarrow h(t) = e^{-\alpha t}$$

where α ($=10^4$ rad/sec) is the analogue cut-off frequency.

(a) Using bilinear transformation, show that the transfer function H(z) is

$$H(z) = \frac{1-a}{2}\left[1 + \frac{(1+a)z^{-1}}{1-az^{-1}}\right] \text{where } a = \frac{2-\alpha T}{2+\alpha T}$$

where $\alpha = 10^4$ rad/sec and the sampling period T is 100 μs.

(i) What is the dc gain of H(z)?
(ii) At what frequency is the H(θ) equal to zero? (θ - digital frequency).
(iii) Calculate the impulse response h(n).
(iv) Assuming that the impulse response decays to 1/e of its initial value at n = N samples, show that :

$$N = \frac{\ln\left\{\frac{\alpha}{\alpha+1}\right\} - 1}{\ln(\alpha)}$$

(b) In order to have the digital cut-off frequency the same as the analogue cut-off frequency α, the analogue frequency must be prewarped. Calculate the prewarped analogue frequency.

[cont'd...]

3. (a) A tranasversal filter has seven filter coefficients b_0 to b_6, with $b_0 = 1$, $b_1 = 2$, $b_2 = 3$ and $b_3 = 4$.

(i) Determine b4 to b6 such that this filter has a linear phase characteristics.
(ii) If all the coefficients (b_0 to b_6) were set to unity, show that the filter has a linear phase characteristics and is given by $\phi(\theta) = -3\theta$. Draw its amplitude characteristics ($|H(\theta)|$ vs θ).

(b) The transfer function for a linear-phase frequency-sampling filter is given by

$$P(z) = \frac{1-z^{-N}}{N}\left[\frac{H_0}{1-z^{-1}} + \left\{\frac{H_1}{1-z^{-1}e^{j2\pi/N}} + \frac{H_{-1}}{1-z^{-1}e^{-j2\pi/N}}\right\}\right]$$

For a realisable filter $H_{-1} = H_1^*$ (i.e. complex conjugate). Assume N =3, $H_0 = 3$ and $H_1 = 3(2 + j\sqrt{3})$.

(i) Draw the frequency-sampling filter structure using delay elements, multipliers and adders.
(ii) Give a filter that has the same frequency response $R(\theta)$, but is realised as an FIR filter.

4. (a) A first-order digital filter is described by the system function

$$H(z) = k\frac{1+bz^{-1}}{1-az^{-1}} \qquad 0 < a, b < 1$$

Assume a = b = 1/2.

(i) Determine k, so that the maximum value of $|H(\theta)|$ is equal to 1.
(ii) Compute the 3-dB bandwidth of the filter H(z).
(iii) Draw a canonic realisation of the system function H(z).

(b) Determine the transfer function of the system shown in Figure 2. Check the stability of the system when r = 0.9 and $\theta_0 = \pi/4$.

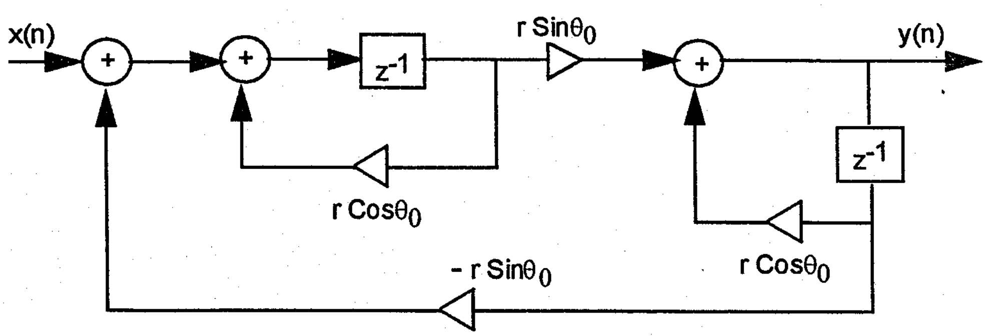

Figure 2

[cont'd...]

5. (a) A signal has been sampled at 44 kHz. For a particular application, this signal is required to have a reduced sampling rate of 16 kHz. Explain how you would achieve this, indicating the type of digital filter which you would use (give reasons).

(b) If the input signal x(n) to the system in Figure 3 is given by

$$x(n) = \frac{1}{2}\delta(n+1) + \delta(n) + \frac{1}{2}\delta(n-1)$$

by writing appropriate equations sketch,

$|X(\theta)|$ against θ and ωT,
$|P(\theta_x)|$, $|R(\theta_x)|$, and $|U(\theta_x)|$ against θ_x and ωT.

Where θ - digital frequency, ω - analogue frequency and T - sampling period.

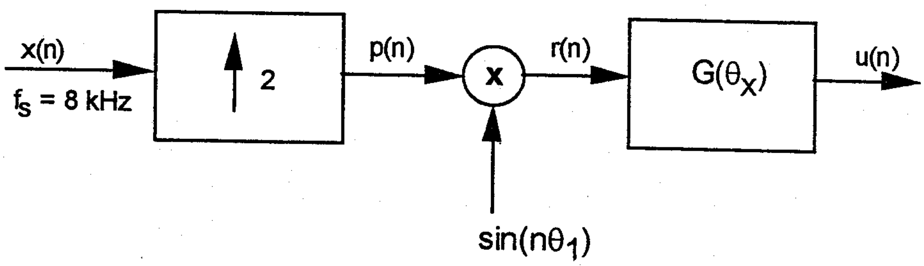

Figure 3

$$\theta = \omega T, \quad \theta_x = \frac{\omega T}{2}, \quad \theta_1 = \frac{\omega_1 T}{2}$$

$$\omega_1 = 2\pi\,(2 \times 10^3)\ rad/\mathrm{sec}$$

$$G(\theta_x) = \begin{cases} \frac{1}{2} & 0 \le |\theta_x| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\theta_x| \le \pi \end{cases}$$

6. (a) In the speech production model, a commonly used approximation for the glottal pulse is as follows:

$$g(n) = na^n, n \ge 0$$
$$g(n) = 0, n < 0$$

Obtain the z-transform, $G(z)$, and hence the frequency response, $G(\theta)$.
Obtain expressions for the magnitude at DC and at half the sampling frequency.

[cont'd...]

(b) Briefly describe the following speech analysis techniques, and suggest applications in which they would be used:

(i) Zero-crossing Count
(ii) Average Magnitude Difference Function
(iii) Autocorrelation function

(c) Explain the basis for Linear Predictive Coding (LPC), and obtain a set of simultaneous equations that form the basis of the autocorrelation and covariance methods of LPC analysis (you are not expected to solve these equations).

7. (a) Show that the optimum coefficient vector for an adaptive FIR filter, e.g. in adaptive system identification applications, is given by the following equation:

$$\boldsymbol{w}_{opt} = \boldsymbol{R}^{-1}\,\boldsymbol{P}$$

where $\boldsymbol{R}$ is the autocorrelation matrix of the input to the adaptive filter, and $\boldsymbol{P}$ is the cross-correlation vector of the filter input and the desired filter output.

(b) With the aid of equations, explain how the LMS adaptive filter algorithm operates. Discuss how the step size determines the convergence behaviour and give an expression for an upper bound on the adaptation step size for convergence.