GX581

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY.

SUMMER EXAMINATIONS

1998/1999

B.E. DEGREE EXAMINATION
INDUSTRIAL ENGINEERING AND INFORMATION SYSTEMS

RELIABILITY AND SAFETY ANALYSIS

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Time allowed: 2 hours
Attempt <u>three</u> questions

Cambridge Elementary Statistical Tables

Q 1 **[20 marks]**

(a) For the exponential distribution, $R(t) = e^{-\lambda t}$

Prove its lack of memory characteristic. [6 marks]

(b) Consider two identical and redundant constant failure rate (CFR) components having a guaranteed life of 2 months and a failure rate of 0.15 failure per year.

What is the system reliability for 10,000 hours of continuous operation? [6 marks]

(c) A component experiences chance CFR failures with an MTTF of 1100 hours. Find the following:

(i) The reliability for a 200-hour mission. [4 marks]
(ii) The reliability for a 200-hour mission if a second, redundant (and independent) component is added. [4 marks]

Q 2 **[20 marks]**

(a) For instantaneous failure rate (or hazard rate), h(t)

$h(t) = -R^1(t)/R(t)$

Prove the reliability,

$$R(t) = \exp\left[-\int_0^t h(t)\,dt\right]$$

[5 marks]

Consequently, show, in Weibull distribution, for

$$h(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}, \quad \theta > 0,\ \beta > 0,\ t \geq 0$$

that $R(t) = \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$

[5 marks]

(b) A turbine blade has demonstrated a Weibull failure pattern with a decreasing failure rate characterised by a shape parameter of 0.6 and a scale parameter of 800 hours.

(i) Compute its reliability for a 100-hour mission. [5 marks]
(ii) If there is a 200-hour burn-in of the blades, what is the reliability for a 100-hour mission? [5 marks]

Q 3 **[20 marks]**

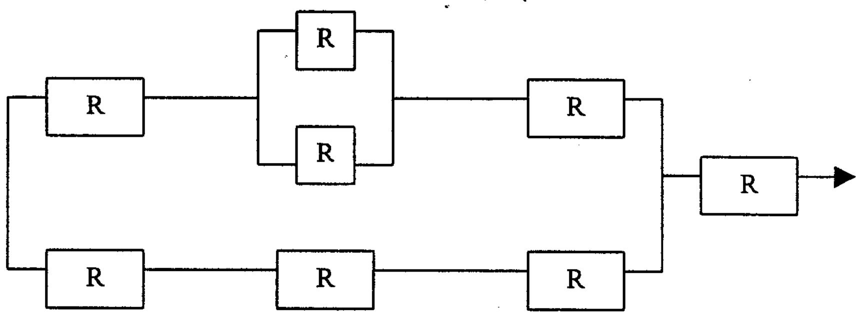
(a) Which system, (i) or (ii) below, has the higher reliability at the end of 100 operating

hours? [10 marks]

(i) Two constant failure rate (CFR) components in parallel each having an MTTF of 1000 hours.
(ii) A Weibull component with a shape parameter of 2 and a characteristic life of 10,000 hours in series with a CFR component with a failure rate of 0.00005.

(b) (i) For the following network, derive an expression for the system reliability in terms of the component reliabilities. Assume that each component has a reliability of R. [8 marks]

(ii) Compute the system reliability if R = 0.9. [4 marks]



Q 4 [20 marks]

(a) Use the following stages in Markov Analysis Methodology to solve a two-component parallel system, which is <u>not</u> subject to repair.

(i) Define all states. [3 marks]
(ii) Construct the rate (state-space) diagram. [3 marks]
(iii) Derive the reliability function. [4 marks]

(b) An engine health monitoring system consists of a primary unit and a standby unit. The MTTF of the primary unit is 1000 operating hours, and the MTTF of the standby unit is 333 hours when in operation and 2000 hours while in standby status.

(i) Estimate the design life of the system, if specifications require a reliability of 0.90. [6 marks]
(ii) What is the system MTTF? [4 marks]

$$R(t) = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-\left(\lambda_1 + \lambda_2^-\right)t}\right]$$

$$\text{MTTF} = \frac{1}{\lambda_1} + \frac{\lambda_1}{\lambda_2\left(\lambda_1 + \lambda_2^-\right)}$$

Q 5 [20 marks]

(a) Derive the equations below for steady-state, interval and instantaneous availability. [5 marks]

(b) Derive the availability equation below for a system with a standby component. [5 marks]

(c) A critical communications relay has a constant failure rate of 0.1 per day. Once it has failed, the mean time to repair is 2.5 days (repair rate is constant).

(i) Compute the steady-state availability. [2 marks]

(ii) Compute the interval availability over a 2-day mission (starting at time zero). [2 marks]

(iii) What is the instantaneous availability at the end of the 2 days? [2 marks]

(iv) If two communications relays operate in parallel, compute the availability in parts (i. to (iii). [2 marks]

(v) If one communications relay operates in a standby mode with no failure in standby, what is the steady-state availability? [2 marks]

$$A_{\text{steady state}} = \frac{r}{r+\lambda}\ ;\ A_{\text{interval}} = \frac{r}{r+\lambda} + \frac{\lambda}{(r+\lambda)^2\,(T_2 - T_1)}\left[e^{-(\lambda+r)T_1} - e^{(\lambda+r)\,T_2}\right]$$

$$A_{\text{instantaneous}} = \frac{r}{r+\lambda} + \frac{\lambda}{r+\lambda}\,e^{-(\lambda+r)t}$$

In standby mode,

$$A = P_1 + P_2,\ \text{where}\ P_1 = \left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1\lambda_2}{r^2}\right]^{-1}\ \text{and}\ P_2 = \frac{\lambda_1}{r}P_1$$