

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

Summer Examinations 1999

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B.Sc. Honours Degree Examination(4BS)

Mathematics Option:
IE428 - 'Quality Control'

Dr. Wright,
Prof. O'Kelly,
Dr. Sheil.

Time allowed: *Two* hours

Attempt *three* questions

Cambridge Statistical Tables supplied.

Mathematical('log') Tables available

Tables for determining Control Chart Parameters appended.

Q.1

State what the following terms mean to you:

(i) *Shewhart Control Chart* (ii) *rational subgroup*

[3 marks]

Differentiate between *probability limits* and *3-sigma limits* for Shewhart charts. [2 marks]

When a measurable characteristic of produced parts is being monitored/controlled, typically two charts, an \bar{X} -chart and an R-chart are operated in tandem. Why? [1 marks]

Draft a general form to be used to record data for, and plot such charts. [3 marks]

The following data was collected from a process manufacturing power supplies. The variable of interest is output voltage; the subgroup size 5. Find centerlines, warning and action *probability* limits for \bar{X} and R charts to control future production. [8 marks]

Subgroup	\bar{X}	R	Subgroup	\bar{X}	R	Subgroup	\bar{X}	R
1	103	4	9	104	3	17	105	3
2	102	5	10	103	5	18	106	5
3	104	2	11	106	4	19	102	2
4	105	11	12	105	3	20	105	4
5	104	4	13	105	4	21	103	2
6	106	3	14	103	2	22	101	4
7	102	7	15	102	3	23	104	5
8	105	2	16	105	4	24	106	4

Q.2

Explain what is meant by the *ARL* of a control chart.

[1 mark]

If P represents the probability that a point will plot inside the control limit(s) on a control chart, show that the corresponding value for the ARL is $1/(1 - P)$ [4 marks]

Under what circumstances would you employ a p-chart?

[1 mark]

A p-chart with control limits at 0.005 and 0.075 is being used to control a process. The chart is based on subgroups of 100 items. When the process is in control, the proportion of nonconforming product is 0.04. [Hint: $Bi(n, p) \rightarrow Poi(np)$]

(i) Find/estimate the probability of false alarms in this case. [2 marks]

(ii) If the process proportion nonconforming "slips" to 0.05, what is the probability that the next point plotted will fall outside the control limits. [2 marks]
Hence determine the ARL to detection of this slip. [1 mark]

(iii) Sketch an OC-curve for this chart. [6 marks]

Q.3

A CUSUM chart is constructed by plotting $S_r = \sum_{i=1}^r (\bar{X}_i - m)$; $r = 1, 2, 3, \dots$

versus r , where the \bar{X}_i 's are sample/subgroup means and m is a fixed reference value.

- (i) Why must care be exercised when choosing m and scales for the plot axes? [1 mark]
 (ii) Explain briefly how to visually interpret CUSUM plots. [2 marks]

Let the subgroup size be n ; the process standard deviation for X be σ and let the chart be scaled so that one unit on the r -axis is equal to σ/\sqrt{n} units on the CUSUM axis.

- (iii) If the current process mean for X is $m + \sigma/\sqrt{n}$, show that "on average" S_r will plot as a line at an angle of 45° to the r -axis. [2 marks]
 (iv) What would be the "average" slope of the plot if the mean was $m + 2\sigma/\sqrt{n}$? [2 marks]

Now, describe briefly how to construct and use a CUSUM key. [2 marks]

Samples of size five were drawn at regular intervals from the output of a production process. The sample averages for a critical parameter were as shown below. Prepare and analyze a CUSUM plot, using the 5-10-10 Rule, if appropriate. [8 marks]

10.45, 10.55, 10.37, 10.64, 10.95, 10.08, 10.50, 10.87, 11.25, 11.46,
 11.39, 11.69, 11.51, 11.28, 11.38, 11.25, 11.63, 11.88, 11.46, 11.67

Q.4

Why undertake Gage Capability Studies? [1 mark]

Define: (i) Gage Repeatability, (ii) Gage Reproducibility, (iii) the P/T Ratio [3 marks]

Two operators twice measure each of 10 parts, using a particular gage. The resulting data is shown below.

Part Number	Operator A		Operator B	
1	50	49	50	48
2	52	52	51	51
3	53	50	54	52
4	49	51	48	50
5	48	49	48	49
6	52	50	52	50
7	51	51	51	50
8	52	50	53	48
9	50	51	51	48
10	47	46	46	47

- (iv) Are both operators consistent in their use of the gage? [3 marks]
 (v) Estimate gage repeatability, gage reproducibility and σ_{gage} . [8 marks]
 (vi) The specification on the dimension measured is 50 ± 10 . Is the gage capable? [2 marks]

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Q5.

State, in the general context of *acceptance sampling* and in your own words, what each of the following acronyms represents:

AQL, RQL/LTPD, AOQ, AOQL.

[4 marks]

- please note that a simple 'expansion' of an acronym to the original words e.g. AOQ to Average Outgoing Quality, does not constitute an acceptable description of what it is that the acronym represents.

Let $P(\theta)$ represent the Operating Characteristic of a single(n,c) attributes sampling plan.

Derive an expression for the AOQ function associated with the plan.

[4 marks]

Under what circumstances does $\theta.P(\theta)$ represent a reasonable approximation for the AOQ function?

[1 mark]

The sampling plan $n=100$, $c=2$ is being used for incoming inspection of lots containing 1000 items.

(i) Sketch the OC-curve for this plan and estimate at what level of lot quality θ , lots will be rejected 90% of the time.

[4 marks]

(ii) Sketch the AOQ function and estimate the AOQL.

[4 marks]