OLLSCOIL NA hÉIREANN
The National University of Ireland

National University of Ireland, Galway

Trinity Examinations, 1998/99

B.E. Degree (Mechanical & Biomedical) Examination

ADVANCED MECHANICAL ANALYSIS AND DESIGN

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Attempt Five Questions
Time Allowed : 3 Hours
Tables 8.1, 9.4 and Figure 11.2 from Burr & Cheatham are attached.

1. Consider the thin tapered bar with circular cross-section subject to an axial force F shown in Figure 1.

(i) Write down an expression for the cross-sectional area A(x) as a function of x, in terms of the end areas A_0 and A_1. (2)

(ii) Assuming that the bar is made from an isotropic linear elastic material, write down expressions for the distributions of axial and tranverse (or radial) strains in the bar as functions of x. You may also assume that the axial stress is uniformly distributed over a cross-section. (3)

(iii) Sketch the strain distributions for the case of $A_1 \rightarrow 0$. (2)

(iv) From (ii) above, derive expressions for the axial displacement as a function of x and the radial displacement as a function of x and the radius r. (8)

(v) Write down an expression for the overall axial strain of the bar and establish a relationship between this and the axial strain distribution. Comment on the resulting relationship. (5)

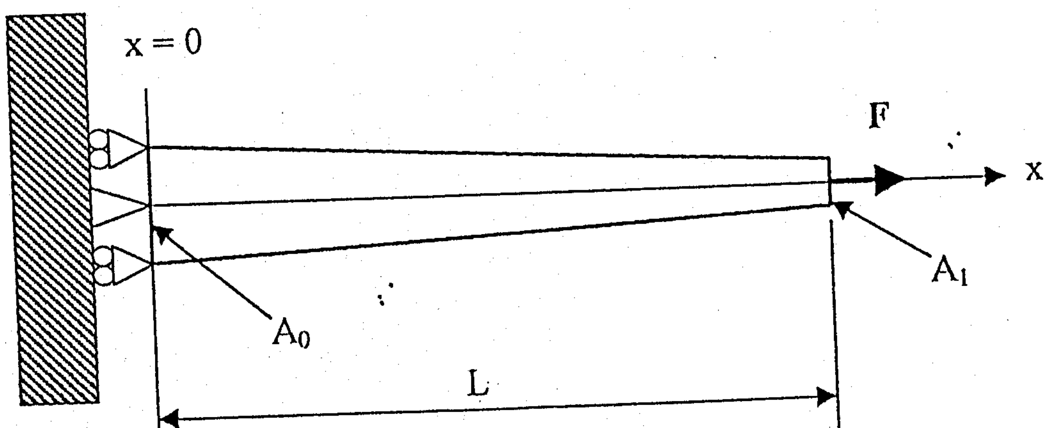

Figure 1

2(a) Write down the membrane equation for thin-walled pressure vessels and define each symbol and term used with the aid of a sketch.

Sketch a free body diagram of an axisymmetric portion of a pressure vessel of thickness t with uniform internal pressure p. Derive the following expressions for σ_m and σ_t :

$$\sigma_m = \frac{pR_t}{2t} \quad , \quad \sigma_t = \sigma_m\left(2 - \frac{R_t}{R_m}\right)$$

(10)

(b) Consider the expansion chamber shown in Figure 2 with a sphere of diameter 500mm, a cylindrical pipe of diameter 100mm and a transition piece of radius $R_m = 75$ mm.

The pressure is 2.0 MPa and the factor of safety is to be 3.0 based on a tensile yield stress of 350 MPa.

Determine the minimum thickness for each of the three segments using the result of (a) and comment on the results.

(10)

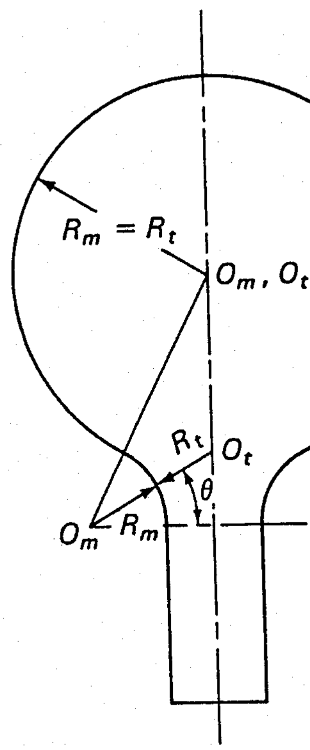

Figure 2

3(a) Consider an interference fit between a solid shaft of radius n and a disk of outer radius q made of the same metal with elastic constants E and υ. Derive the following relationship between the interface pressure P_f and the diametrical interference Δ :

$$\frac{4P_f n q^2}{E(q^2 - n^2)} = \Delta$$

Write down expressions for the radial and tangential stresses as functions of the radius in the assembly and draw a diagram showing the stress distributions. Comment on the resulting distributions.

(12)

(b) Consider a gear box intended for transmitting torque in low speed applications. A gear with a tooth root diameter of 240 mm and 20 mm thickness is assembled onto a shaft of 100 mm diameter with a diametrical interference of 0.12 mm.

Calculate the fit pressure and the maximum torque that can be transmitted by the joint given a friction coefficient of 0.2. Assume all components are made of steel with a Young's modulus of 207 GPa.

(8)

4. Consider the cantilever beam shown in Figure 4. It is supported at the free end by a spring of stiffness k.

(i) Establish whether or not this is a statically determinate problem. **(2)**

(ii) Determine the deflection of the beam at the spring and the bending moment at the built-in end.

(14)

(iii) Determine the bending moment at the built-in end if the spring support is replaced by a pin joint.
In this case does the moment depend on the elastic properties of the beam ?

(4)

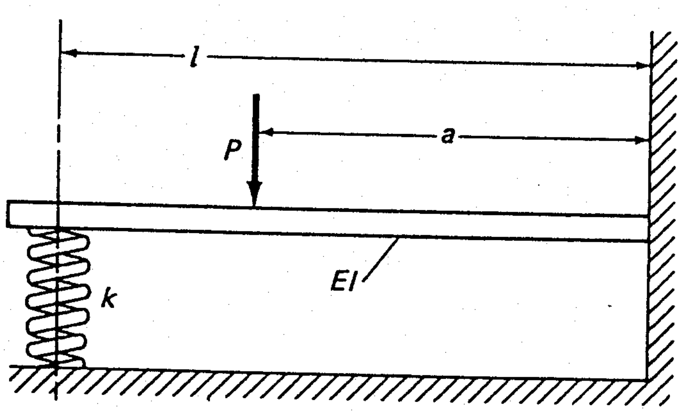

Figure 4

5. A coil spring shown in Figure 5 has a rigid rod brazed to it. A force P is applied to the rod at point C coinciding with the centre of the coil at an angle α. Consider that the coil extends for 360°. Determine the deflection of the point C in the direction of the force and perpendicular to the direction of the force for the following cases :

(i) $\alpha = 0$
(ii) $\alpha = 90°$

What interesting property of the spring is revealed ? **(20)**

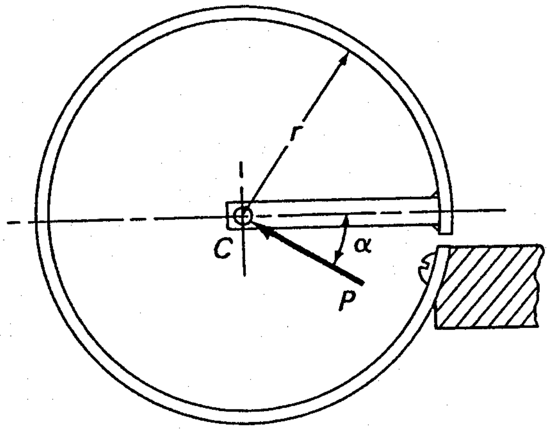

Figure 5

6. A dc motor delivers a torque of 400 Nm at its lowest speed. The motor is fastened to a vertical wall as shown in Figure 6. In its base are holes for six identical bolts, two in each of three rows, at locations a, b and c in Figure 6. The gravity force due to the motor is 2000 N and its centre of gravity is 200 mm from the wall.

For the worst case where the bolts must support the motor torque and the gravity moment, determine the forces in the bolts (neglecting pre-tightening). Derive all formulae used. **(20)**

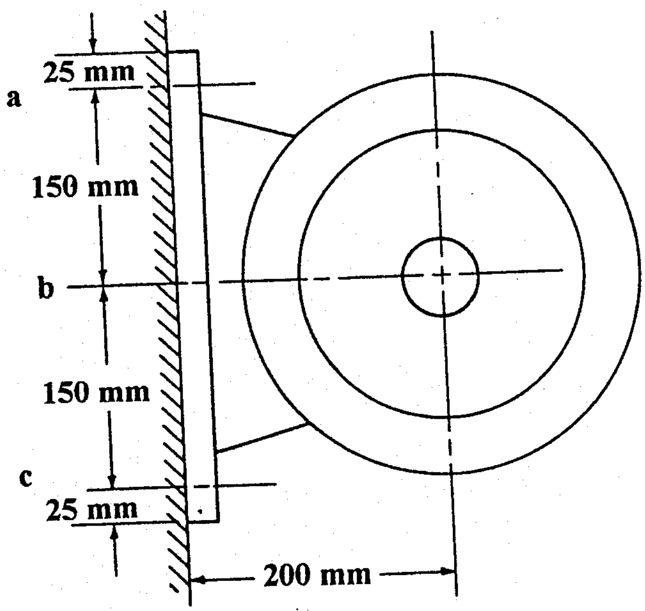

Figure 6

7. The ball end of a steel link is 10 mm in diameter. It is socketed in the hard-bronze bearing-alloy spherical seat, 10.1 mm in diameter, of a rocker arm in a valve-operating linkage as shown in Figure 7. The maximum pressure between the ball and its seat is limited to 200 MPa.

For hard bronze, E = 110 GPa, $\nu = 0.3$, and for steel, E = 200 GPa, $\nu = 0.3$

Determine the maximum allowable load on the ball and socket such that the limiting pressure is not exceeded.

(20)

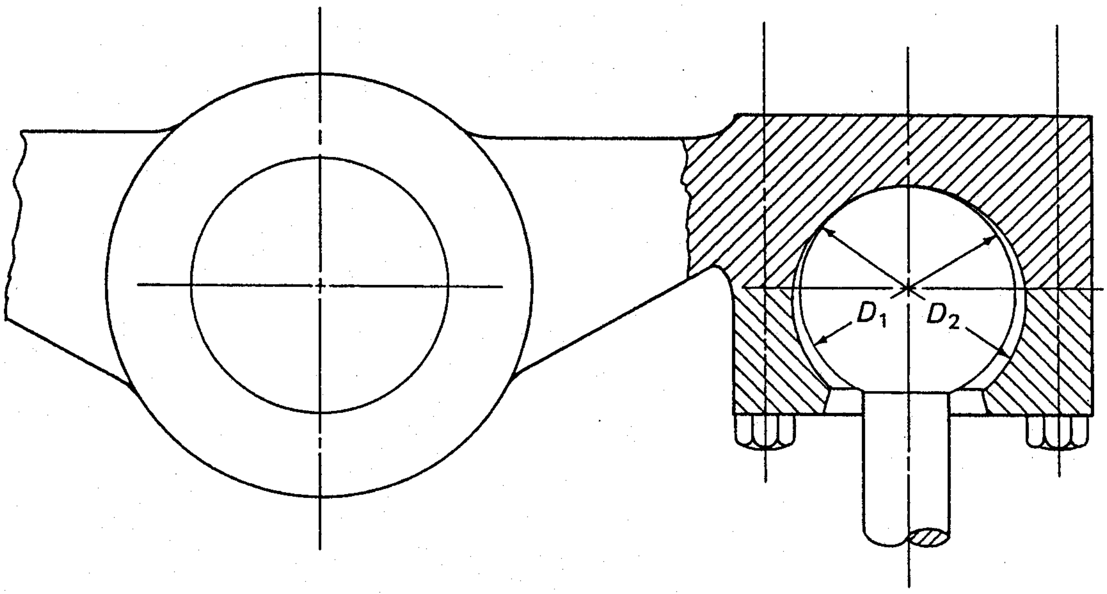

Figure 7