OLLSCOIL NA hÉIREANN
The National University of Ireland

National University of Ireland, Galway

Trinity Examinations, 1998/99.

M.Eng.Sc. Examination (Mechanical)

MECHANICS OF COMPOSITE MATERIALS

Professor J.J. O' Connor
Professor P.J. Nolan
Dr. C. Ó Brádaigh

Attempt Question 1 & Two other Questions
Question 1 worth 20 marks.
All Other Questions worth 40 marks each
Time Allowed: 3 Hrs.

The following are available :
Copies of Equations for : [Q] in Terms of Eng. Constants, Expansional Stress Resultants, Stiffness Transformations, Compliance Transformations, Beam Equations, Transformation of Stress and Strain, Tsai-Wu Failure Theory.

1(a) By expansion of the energy density, U, for an elastic body, derive the generalised form of Hooke's law :

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \quad (i, j, k, l = 1, 2, 3)$$

Invoke material symmetry to reduce this expression for orthotropic and transversely isotropic materials. **(20)**

2(a) Derive the following expressions for a transversely isotropic composite laminated plate :

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^\circ \\ \kappa \end{bmatrix}$$

where $[\varepsilon^\circ]$ are the mid-plane strains and $[\kappa]$ are the midplane curvatures. Clearly state all assumptions employed. Identify the coupling terms and sketch their coupling modes. **(20)**

(b) Calculate the effective laminate constants for a $[0°/\pm 40°]_s$ laminate of ultra-high modulus CF/epoxy. **(20)**

Given : E_L = 250 GPa , E_T = 20.2 GPa

G_{LT} = 7.0 GPa , ν_{LT} = 0.29

3(a) Derive the rule of mixtures expressions for the thermal expansion coefficients α_L and α_T for a continuous fibre reinforced composite ply, given the mechanical and thermal expansion properties of the fibre and matrix, as well as the fibre volume fraction. Clearly state all assumptions employed. **(20)**

(b) A Kevlar/Epoxy $[0°/90°]_s$ laminate is cooled from its processing temperature of 220°C to room temperature (20°C) in an unrestrained manner. Calculate the residual stresses and strains in the 0° plies. **(20)**

Given : E_L = 90 GPa , E_T = 10.2 GPa

G_{LT} = 3.1 GPa , ν_{LT} = 0.34

α_L = 2.1×10^{-6}/°C

α_T = 22.0×10^{-6}/°C

4. A [0°/90°/0°] graphite epoxy laminate is tested to failure in a tensile test in the 0° direction.

(a) Apply the Tsai-Wu failure criterion to predict the laminate stress at which first-ply-failure occurs. **(20)**

(b) Assuming a slow unloading of the 90° plies, with a tangent modulus of E_T = -3.4 GPa, carry out a progressive ply failure analysis and plot the stress-strain curve for the test. **(12)**

(c) What is the stress-state in each ply at first-ply and final-ply failure ? **(8)**

Given : E_L = 150 GPa , E_T = 8.0 GPa

G_{LT} = 2.3 GPa , ν_{LT} = 0.30

$X_1^C = X_1^T$ = 1720 MPa

X_2^T = 49 MPa

X_2^C = 95 MPa

X_6 = 100 MPa

5(a) Derive the following relationship between composite beam modulus and central deflection in a 3-point flexure test.

$$E_x^b = \frac{PL^3}{4bh^3 W_c}$$

where b, h and L are the beam width, thickness and length respectively. **(15)**

__Note__ : Some beam equations are given on attached sheet.

(b) A load of 500 N is applied to a [90°/0°/0°/90°] S-glass/epoxy beam in a 3-point bend test. The beam dimensions are L = 60 mm, h = 2.4 mm, b = 12.5 mm.
Calculate the maximum stress and its location.

(15)

Given : E_L = 55 GPa , E_T = 14 GPa

G_{LT} = 3.4 GPa , ν_{LT} = 0.3

(c) What load will cause first–ply failure ? Use the maximum strain failure criterion.

$Y_1^C = Y_1^T = 1.9\,\%$

$Y_2^T = 0.4\,\%$

$Y_2^C = 1.2\,\%$

(10)