OLLSCOIL NA hÉIREANN
The National University of Ireland

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Second Year Mechanical Engineering Examination

INSTRUMENTATION

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Attempt *Five* Questions.
Time Allowed: *3 Hrs.*
Graph Paper is available.

The following are attached :
Equations and diagrams for First-Order Instrument (step response), Second Order Instrument (step response), Sensor Equations, Differential Amplifier and Linear Regression.

1. You wish to measure displacements with a potentiometer. Answer the following questions concerning a potentiometer.

   (a) Draw the basic circuit diagram for a potentiometer, and write the transfer function. What is the frequency response of this instrument?

   (6)

   (b) In using a potentiometer, non-linearity can be caused by the measuring device. Using a circuit diagram, explain this phenomenon. How could an op-amp improve the linearity?

   (7)

   (c) Given the potentiometer has a static sensitivity of 2-V/inch, and a full scale output of 10 V, find the independent non-linearity from the data given below:

Input (inch):	0.5	1.00	2.00	4.00	5.00
Output (V):	0.9	2.05	3.95	8.00	9.95

   (7)

2(a) The dynamic response of a second-order instrument is governed by the following second-order differential equation:

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

Derive the transfer function given below for a second-order instrument. Define the physical constants (ω_n, K, ζ) based on the constants in the second-order differential equation.

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{\left(\frac{j\omega}{\omega_n}\right)^2 + \left(\frac{2\zeta}{\omega_n} j\omega\right) + 1}$$

(10)

(b) A pressure transducer yields the voltage output shown in Figure 2 for a step input of pressure (assume K = 10). Determine the following characteristics of the pressure transducer and sketch the bode plot labeling the numerical value of ω_n, y-intercept, and slope:

i. damping ratio, ζ
ii. damped natural frequency, ω_d
iii. natural frequency, ω_n

(10)

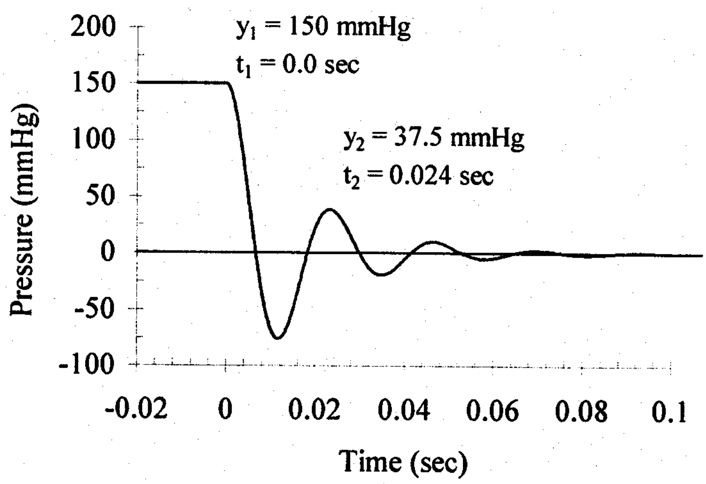


Figure 2

3. You are required to measure pressure oscillations using a capacitance microphone and a voltage amplifier. The capacitor has a surface area of 1 cm^2, and an initial capacitance of 40 pF. The excitation voltage (E) is 5 V. The instrument must be able to sense displacements up to 5 μm without saturating the amplifier, and have a frequency response above 30 Hz with no more than a 5% loss in the input amplitude. The voltage amplifier is to have the maximal gain possible without exceeding the typical linear output range of an op-amp.

(a) Design the above instrument. Draw the final instrument circuit and label the numerical value of all components. **(10)**

(b) Determine the transfer function for the instrument and sketch the bode plot. Label the numerical value of the cut-off frequency, y-intercept, and slope. **(5)**

(c) In making measurements, you find that you have high frequency noise. Determine the capacitor size that must be added to the voltage amplifier to attenuate frequencies above 1 kHz. Determine the new transfer function. **(5)**

4. You need to construct a preamplifier for measuring an electrocardiogram. The preamplifier must have a total gain of 1000, with a 3 op-amp differential gain of 25 and a non-inverting, low-pass filter gain of 40 (total gain = 25x40 = 1000). The frequency range is 0.05 – 150 Hz.

(a) Draw a normal ECG waveform. Label the peaks and state their origin. **(5)**

(b) What are five basic requirements for a biopotential amplifier? **(5)**

(c) Design a preamplifier to meet the specifications given above. Assume $R_1 = R_3 = 10$ kΩ for the differential amplifier (see Schematic of a Differential Amplifier on p.5), $R_f = 150$ kΩ for the non-inverting, low-pass filter, and the resistor in the high-pass filter is 3 MΩ. Draw the final circuit and give the numerical values for all of the components. **(10)**

5. Answer the following questions on flowmeters.

(a) Describe the principle by which an electromagnetic flowmeter operates. **(5)**

(b) Discuss how geometry, blood properties, and velocity profile can cause errors in the use of electromagnetic flowmeters. **(5)**

(c) Describe the principle by which a transit-time ultrasonic flowmeter operates. **(5)**

(d) Compare and contrast the ultrasonic flowmeter to the electromagnetic flowmeter. **(5)**

6. Answer the following questions on sensors and their dynamic responses :

(a) You are asked to use a thermocouple to measure the temperature of a water bath. After submersion in the water bath, the thermocouple output is 1.79 mV with the reference junction held at 25 °C. Calculate the temperature of the bath and state the thermocouple law that allows the calculation. The calibration table for the thermocouple is give below.

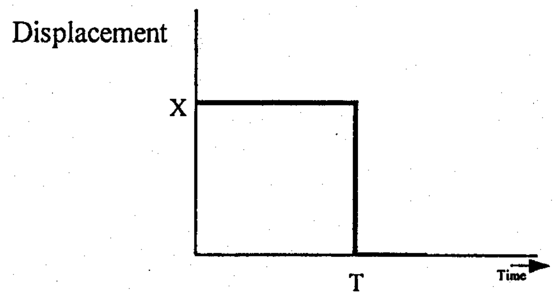
Calibration Table for Thermocouple (reference junction @ 0 °C)

Temperature (°C)	E (mV)
0	0.15
25	0.71
38	1.94
66	2.26
72	2.50
80	2.62

(5)

(b) Draw the piezoelectric sensor output voltage response to the step displacement (X) shown below, and state how you can improve the dc response of a piezoelectric sensor:

(5)



(c) You need to measure an oscillating linear displacement. You have an LVDT in the laboratory, but you do not know the time constant of the sensor. Due to your wonderful instrument knowledge, you decide to input a step displacement of 10 cm, and record the sensor output (Y) vs. time in order to determine the time constant. The following data was recorded:

Time (msec)	Y(cm)
0	9.9
1	9.5
4	8.0
11	5.8
40	0.9

Calculate the time constant using linear regression.

(10)

Useful Equations and Diagrams

First-Order Instrument (step response)

Low-pass filter: $y(t) = A(1 - e^{-t/\tau})$

High-pass filter: $y(t) = Ae^{-t/\tau}$

Second-Order Instrument (step response)

Overdamped, $\zeta > 1$:

$$y(t) = -\frac{\zeta + \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} Ae^{\left(-\zeta + \left(\sqrt{\zeta^2 - 1}\right)\varpi_n t\right)} + \frac{\zeta - \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} Ae^{\left(-\zeta - \left(\sqrt{\zeta^2 - 1}\right)\varpi_n t\right)} + A$$

Critically damped, $\zeta = 1$: $y(t) = -(1 + \varpi_n t)Ae^{-\varpi_n t} + A$

Underdamped, $\zeta < 1$:

$$y(t) = -\frac{e^{-\zeta\varpi_n t}}{\sqrt{1-\zeta^2}} A\sin\left[\left(\sqrt{1-\zeta^2}\right)\varpi_n t + \phi\right] + A$$

$$\phi = \arcsin\sqrt{1-\zeta^2}$$

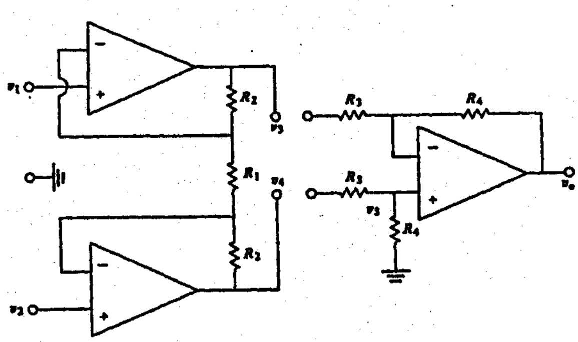
Damped natural frequency: $\varpi_d = \varpi_n\sqrt{1-\zeta^2}$

Sensor Equations

Capacitance Sensor: $C_o = \left(8.854x10^{-12}\right)\frac{A}{x_o}$ (A = [m^2] and x = [m])

Differential Amplifier

Gain: $G_d = \frac{R_4}{R_3}\left(\frac{2R_2 + R_1}{R_1}\right)$



Schematic of a 3 op-amp differential amplifier

Linear Regression

Slope:

$$m = \frac{n\sum x_d y - \left(\sum x_d\right)\left(\sum y\right)}{n\sum x_d^2 - \left(\sum x_d\right)^2}$$

y-intercept:

$$b = \frac{\left(\sum y\right)\left(\sum x_d^2\right) - \left(\sum x_d y\right)\left(\sum x_d\right)}{n\sum x_d^2 - \left(\sum x_d\right)^2}$$