OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SPRING EXAMINATIONS, 1999

THIRD ENGINEERING (CIVIL) EXAMINATION

MECHANICS OF FLUIDS (EH 301)

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Attempt *any* **five** questions: (Time: 3 hours)

(The Handout of Tables and Diagrams provided may be useful in one or more questions)

Section A

1.(a) In the context of fluid flow, what *(very briefly!)* do you understand, both *physically* and *mathematically*, by the expression "**the divergence of the velocity vector** V, **(Div** $\underline{V}$ **), at a point in the flowfield**"?

(b) **Derive,** *from first principles*, the general "**conservation of mass**" equation

$$\frac{\partial C}{\partial t} = -(\underline{\nabla} \cdot \underline{M})$$

for a *point* in the flowfield, where t is time, the scalar C can be *either* the fluid *or* the solute mass concentration, M is the vector of the corresponding *mass flux per unit area* at the point and $\underline{\nabla}$ is the "grad" (i.e. Nabla) vector.

(c) For the case where C denotes the **fluid mass concentration** (i.e. the fluid mass density ρ), show that the **continuity equation** of flow for a **compressible fluid** is given by

$$\text{Div}\,\underline{V} = -\frac{1}{\rho}\left[\frac{\partial \rho}{\partial t} + (\underline{V}.\underline{\nabla})\rho\right] = -\frac{1}{\rho}\frac{d\rho}{dt}$$

(d) Hence, derive the corresponding continuity equation for a point in the flowfield when the **fluid is incompressible.**

(e) For the case where C denotes the **solute mass concentration** c, (rather than the *fluid* mass density ρ), assuming the validity of **Fick's first law of diffusion,** given in *vector* form as

$$\underline{M} = -D\underline{\nabla}c$$

where D is the **molecular diffusivity coefficient** (which is considered as a scalar for *laminar* flow), show that the **conservation of mass** equation has the form

$$\text{Div}\,\underline{V} = -\frac{1}{c}\frac{dc}{dt} + \left(\frac{D}{c}\right)\underline{\nabla}^2 c$$

known as the **advective-Fickian diffusion equation** for *laminar* flow of a **compressible fluid.**

(f) Hence, derive the corresponding **advective - Fickian diffusion equation** for the case of *laminar* flow of an **incompressible fluid.**

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2. (a) Distinguish *clearly (but very briefly)* between

a solid	and	**a fluid**
a linear elastic solid	and	**a Newtonian fluid**
a real fluid	and	**an "ideal" fluid**
the "rate of strain" tensor	and	**the" rotation" tensor**
a "potential" flowfield	and	one that is otherwise.

(b) Show that the **"rate of change of velocity tensor"** $[\nabla V]$ at any point in a flowfield can be *uniquely* decomposed into the **"rate of strain tensor"** [E] and the **"rotation tensor"** [R] at that point.

(c) **Including** consideration of the **origin**, demonstrate whether or not the flowfields A and B defined below are truly **"potential"**.

(A) $v_r = 0$; $v_\theta = \omega' r$, (in **polar form**, with parameter $\omega' =$ constant)

(B) $u = ax$; $v = -ay$, (in **cartesian form**, with parameter $a =$ constant).

(d) Show that, for **flowfield A** above, $[\nabla V] = [R]$, which is **constant** throughout.

(e) Show that, for **flowfield B** above, $[\nabla V] = [E]$, which is **constant** throughout.

(f) Identify the **physical analogies** of the flowfields A and B defined above.

(g) *Very briefly*, why are these **tensors** of such fundamental importance in the theory of flow of **real Newtonian fluids**?

3. (a) In the context of a **2-dimensional "potential" flowfield**, explain *clearly* the basis of **Rankine's method for the graphical superposition of flownets.**

(b) Demonstrate the **Rankine method** by plotting, to a suitable scale (*clearly indicated*), the flow pattern (*streamlines* only) corresponding to the **combination** of

a **source** of discharge $Q_1 =$ 72m²/s/m depth located at the point (-6,0)

and a **sink** of discharge $Q_2 = -$ 72m²/s/m depth located at the point (+6,0)

using a stream function increment $\delta\psi = \delta Q = 2$ m²/s/m depth.

(c) Write down the **general equation** for a **streamline** passing through an arbitrary point P in the above **combined flowfield.**

(d) Show *mathematically* that **all streamlines in the combined flowfield** are **circles**, each having the straight line joining the origins of the source and the sink as the **common chord.**

(e) If the **source and sink each approach the origin,** in such a manner that, as the distance δs between them approaches zero, the product $(\frac{Q}{2\pi} \times \delta s)$ approaches a constant C in the limit, **what kind of flow net will this produce?**

4.(a) In the context of **potential flow theory,** what (*very briefly!*) do you understand by *each* of the following terms:

potential flowfield, **stream function,**
velocity potential function and **flow net ?**

(b) Establish the **analogy of potential flow with that of laminar flow of a Newtonian fluid through a porous medium,** defined by the **Darcy equation** for the velocity vector $\underline{V}$, of the form

$$\underline{V} = -K\underline{\nabla}h$$

where h is the piezometric head and K is the transmission constant.

(c) Assume that the **flow net configuration,** for the transverse section of a long impervious dam set in a horizontal porous layer of granular non-cohesive material overlying a horizontal impervious boundary, **as is shown** in the accompanying figure No. 1;

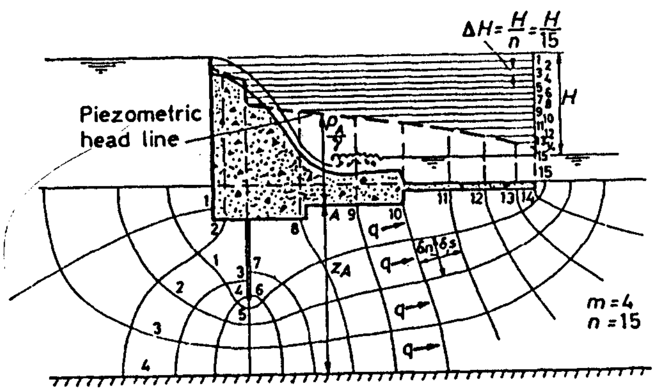

Fig. No. 1 The *given* flownet for the porous medium under the impervious concrete dam, the porous layer overlying a horizontal impervious layer.

The transmission constant K for the medium of the porous layer is estimated at 2.0m per day and it is assumed that water percolates under the dam, uniformly along its length, in accordance with **Darcy's law.**

(i) If the static water level head difference **H,** across the dam, is 30.0 m, **estimate the flow rate** of seepage **Q** under the dam, per metre length of dam, for the **given flow-net.**

(ii) Indicate *briefly* how the **flownet** may also be used to construct the piezometric head line above the dam section and hence to estimate the **resultant thrust** exerted by the seepage flowfield on the base of the dam.

5.(a) Explain *(very briefly !)* each term of the *three-dimensional* **vector form** of the **Navier Stokes dynamic equation**

$$\frac{d\underline{V}}{dt} = \frac{\partial \underline{V}}{\partial t} + (\underline{V}\cdot\underline{\nabla})\underline{V} = \frac{\partial \underline{V}}{\partial t} + [\nabla V]\underline{V} = -\frac{1}{\rho}\underline{\nabla}(P_h + \rho g h) + \nu \underline{\nabla}^2\underline{V} + \frac{\nu}{3}\underline{\nabla}(\underline{\nabla}\cdot\underline{V})$$

where $\nu = \dfrac{\mu}{\rho}$ = the kinematic viscosity of the fluid, ρ being the fluid density.

(b) State *clearly* **(i) the most general conditions of flow** and

(ii) the type of fluid for which the **Navier Stokes** equation is valid.

(c) Using the **Navier-Stokes equation,** or *otherwise,* and stating clearly all of the **boundary conditions** of your solution, **outline the derivation of the Haugen-Poiseuille equation**

$$h_L = \frac{32\mu V_{av} L}{\rho g D^2}$$

for the **friction** head loss due to **laminar** flow of a virtually **incompressible fluid in a straight pipe** of constant section, where the pipe is flowing full.

(d) Is it *necessary* to derive the **Haugen-Poiseuille equation** from the **Navier-Stokes equation**? *Briefly,* explain your answer!

6.(a) *Outline* the derivation of **Prandtl's laminar/turbulent transition equation**

$$C_{f,}\text{total} = \frac{F_{L,total}}{\frac{1}{2}\rho U^2 L} = \left[\frac{0.074}{(Re)_L^{1/5}} - \frac{1743}{(Re)_L}\right]$$

for the coefficient of plate friction C_f, involving **both laminar and turbulent boundary layers** acting **on a "hydraulically smooth" thin flat plate** at zero angle of incidence to the direction of flow.

(b) What form of **transition in the thickness of the boundary layer** at the **critical length** is implied in the derivation of **Prandtl's transition equation** ?

(c) A pontoon, 6 m in length (= L) and 3 m in width, with vertical sides, floats to a depth D of $0\cdot5$m. The pontoon is towed in *sea water* at 10°C (for which the density ρ_{sw} = 1024 kgm^{-3} and dynamic viscosity $\mu_{sw} = 1\cdot31\times10^{-3}$ Nsm^{-2}) at a *constant* velocity of $U = 2$ ms^{-1}. If the *critical* plate Reynold's number is taken as $(Re)_{x\ critical} = 5\times10^5$ and if the *form drag* and all wind and sea currents are neglected, **estimate**

(i) the **total skin friction drag force** acting on the pontoon,

(ii) **the thickness** δ_L **of the boundary layer** at the **trailing end of the** pontoon, i.e. at $x = L$,

(iii) **the boundary shear stress** $\tau_0(L)$, i.e. at distance $x = L$.

(iv) **the mean boundary shear stress** $\overline{\tau_0(L)}$ **over the entire length** L.

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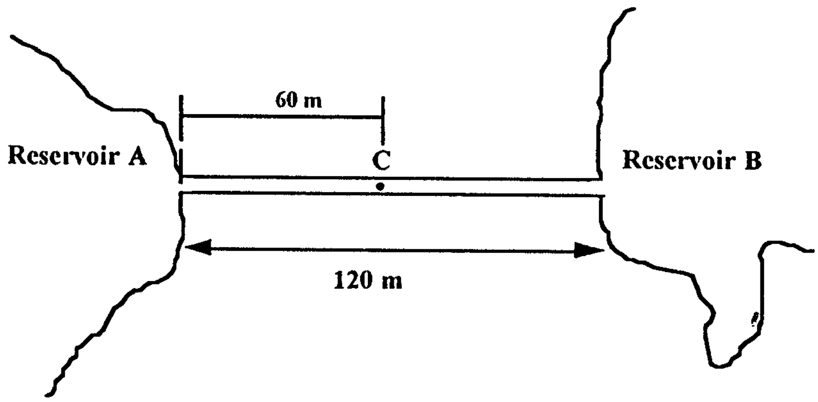
Section B

7. Derive the equation of motion in the x-direction for a viscous fluid using a cubical control volume. Express the equation of motion in the x - direction for an inviscid flow field. Indicate how the Navier Stokes equation for laminar flow is obtained from the equation of motion in the x - direction.

8. A service canal connects two large reservoirs. At point C midway between the two reservoirs an incident occurs in which an instantaneous slug of pollutant is released causing the concentration at C to immediately rise to 20 mg/l. The connecting reservoirs are very large and the pollutant concentration in these reservoirs can be assumed to be constant over time at a background concentration of 1.0 mg/l. The flow in the service canal is negligible.



(a) Given that the objective is to model the spread of the pollutant in the service canal, express the governing equation for this problem and detail any assumptions made.

(b) Using a finite difference solution scheme present the above governing equation in finite difference form.

(c) Using a time step Δt = 30sec, a spatial step Δx = 20m and a dispersion coefficient of 4 m^2/s :

(i) Construct the finite difference domain

(ii) Present the initial conditions and the boundary conditions for this problem.

(iii) Compute the pollutant concentrations at each node for one time step.

9. Describe in detail the various steps involved in conducting a water quality numerical modelling study of an industrial outfall located in an estuarine water body.