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SEMESTER 2 (SUMMER) EXAMINATIONS 1998-99

Applied Physics & Electronics – Paper 2
Physics & Computing – Paper 2

EP324: Signal Analysis

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Answer THREE questions	Time allowed: TWO hours

Q.1 Identify the following equations and explain their significance. Define and relate the terms T and ω_o as used in the equations.

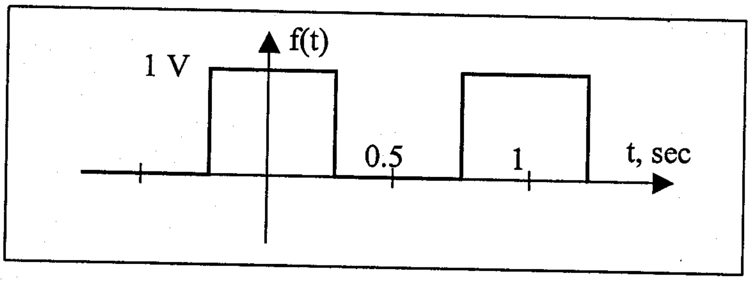
$$f(t) = \sum_{n=-\infty}^{n=\infty} c_n \exp(jn\omega_0 t) \qquad c_n = \frac{1}{T}\int_T f(t)\exp(-jn\omega_0 t)\,dt$$

Show that the Complex Fourier Series, for the 1 V amplitude square wave f(t) shown in the diagram, is given by

$c_0 = 0.5,$

and, for $n \neq 0$, by

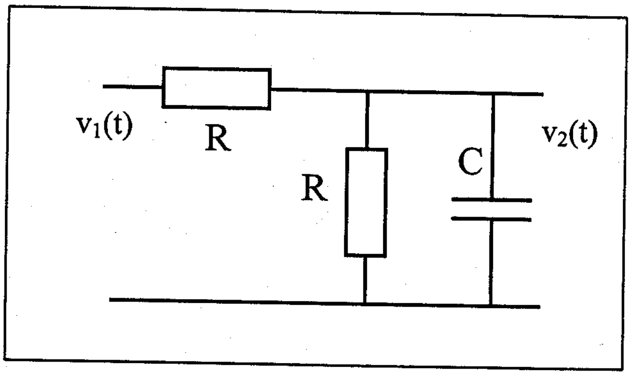
$c_n = \sin(n\pi/2)/(n\pi).$



Make an approximate sketch of the power spectrum of f(t). Calculate the total power in f(t), and the fraction of the total power contained in the DC term plus first harmonic.

Q.2 What is the main physical significance of the Fourier transform of a waveform f(t)? Explain briefly how the Laplace Transform may be developed as an extension of the Fourier Transform, for waveforms whose Fourier Transform does not exist.

Find the transfer function H(s) of the network shown opposite, if R = 1 Ω, C = 1 F. Calculate the network response, $v_2(t)$, to an input $v_1(t)$ = u(t), where u(t) is the unit voltage step function. Identify the *forced*, *natural*, *transient*, and *steady state* components in the solution.



Note the following Laplace Transform pairs:

f(t)	u(t)	u(t)exp(-αt)
F(s)	1/s	1/(s+α)

p.t.o.

Q.3 Say what is meant by a *LTI* system. Identify the nature of the LTI system with the transfer function H(s) shown opposite, where $\zeta < 1$. Define the system *Q-value*, *damping factor*, *natural frequency* and *damped frequency*, relating them to the terms shown in H(s).

$$\boxed{H(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}}}$$

Sketch the general nature of (a) the frequency response and (b) the step function response, of such a system.

A LTI system has 2 poles at s = -1 ± 10j. Calculate its Q-value and damped frequency.

Q.4 Say briefly what is meant by the process of *discretization* of a CT (Continuous Time) system to a DT (Discrete Time) equivalent. State the fundamental problems inherent in this process.

A CT system is described by the differential equation $\frac{dy}{dt} + 2y = 5x$, where y(t) is the system output and x(t) is its input. Use the Backward Euler Algorithm to discretize this equation, for a sampling interval T, and show that the resulting DT difference equation is:

$$y[n+1] = \frac{y[n] + 5Tx[n+1]}{1+2T}$$

Calculate the numerical values of the first 3 terms in the system impulse response h[n], for T = 0.1 second. Can this DT system ever become unstable for any choice of T? Justify your answer.

Q.5 Answer any *two* of the following:

(a) Explain what are meant by *cross correlation* and *auto correlation* functions, and describe how they are evaluated. What is the relationship between the auto correlation, R(τ), and power spectral density, psd, for a signal? Explain how auto correlation may be used to detect the presence of a periodic signal buried in noise.

(b) Give a full account of the *Gibb's phenomenon* in Fourier Series work, carefully explaining how it arises. Describe how the use of a *window function* in truncating a Fourier Series can reduce the Gibb's effect.

(c) State the general form of Mason's Rule for the transfer function of a system represented in block diagram form.

Use Mason's Rule to find the overall transfer function H(s) of the system shown in the diagram opposite, where G1 = 1, G2 = 1/(s+1), H1 = s/(s+1), and H2 = 1/s. Find the positions of the poles and zeros in H(s).