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NATIONAL UNIVERSITY OF IRELAND
GALWAY

SEMESTER 2 (SUMMER) EXAMINATIONS 1998-99

Experimental Physics – Paper 1

EP314: Thermodynamics and Vacuum Physics

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Answer THREE questions	Time allowed: TWO hours

Q.1 (a) For a system in contact with a heat bath what is the probability that the system:

1. Is in a state r with energy ε_r?
2. Has energy ε_r?

Define all the terms used in the above.

(b) Write down the general form for the average energy of a system in contact with a heat bath. Show that the average energy can be written as

$$<E> = -\partial \ln Z/\partial\beta.$$

(c) (i) What is a Schottky defect?

(ii) Let the energy of formation of a Schottky defect be ε, and consider a crystal of N atoms with n defects. If the system is in contact with a heat bath at temperature T find:

1. The average energy $<E>$ of the system.
2. The average number of defects as a function of temperature.
3. Given that the entropy $S = k_B \ln Z + <E>/T$ find the Helmholtz free energy F.

Q.2 (a) (i) Define the isobaric expansion coefficient β_0.

(ii) What is the third law of thermodynamics?

(iii) Using the fact that $dG = - SdT + VdP$ show that, $(\partial S/\partial P)_T = -(\partial V/\partial T)_P$ (third Maxwell relation). Use this to show that

$$\lim_{T\to 0} \beta_0 = 0.$$

(b) A solid contains N magnetic atoms each with spin 1/2. At sufficiently high temperatures, the spins are randomly oriented, while at low temperatures the interactions between the magnetic atoms causes them to exhibit ferromagnetism, with the result that their spins become oriented along the same direction as $T \to 0$. A very crude approximation suggests that the spin-dependent contribution $C(T)$ to the heat capacity of this solid has the approximate temperature dependence given by

$$C(T) = C_1(2T/T_1 - 1) \qquad \text{if } T_1/2 < T < T_1$$
$$= 0 \qquad \text{otherwise}$$

Use entropy considerations to find an explicit expression for the maximum value C_1 of the heat capacity.

p.t.o.

Q.3 (a) In our derivation in class for the entropy of an ideal gas, in order to obtain the correct formula for S, we had to incorporate two steps which did not follow from classical mechanics. What were these steps? Explain how these steps showed glimpses of deficiencies in classical mechanics, which would be rectified by quantum mechanics.

(b) Consider an ideal gas of particles in the extreme relativistic range; i.e., the kinetic energy E of a particle and its momentum $\mathbf{p}$ are related by $E = c|\mathbf{p}|$. Find:

1. the partition function;
2. the Helmholtz free energy;
3. the equation of state, the entropy (and show that it is extensive);
4. the internal energy, and the heat capacity at constant volume for this gas.

Hint: use spherical polar co-ordinates when evaluating the partition function, i.e., $\int_{-\infty}^{\infty} d^3\mathbf{p} = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \sin\theta \, d\theta \, d\phi \, p^2 dp$, where $p = |\mathbf{p}|$.
Also, you may need the integral $\int_0^{\infty} x^2 \exp(-\alpha x) \, dx = 2/\alpha^3$, where $\alpha > 0$.

Q.4 (a) Show that the complete conversion of heat into work in a heat engine would violate the second law of thermodynamics.

(b) (i) The Brayton cycle consists of an isentropic compression, a isobaric expansion, an isentropic expansion followed by an isobaric compression to complete the cycle. Let T_1 and T_2 be the temperatures at the start and end of the isentropic compression; and let T_3 and T_4 be the temperatures at the start and end of the isentropic expansion. Plot the four stages on a T-S diagram and on a P-V diagram. The Brayton cycle is the ideal gas cycle for what type of heat engine?

(ii) For an ideal gas undergoing an isentropic expansion how are the temperatures and pressures before and after the expansion related? From this, show that, for the Brayton cycle, $(T_4/T_1) = (T_3/T_2)$ and $(T_1/T_2) = r_p^{(1-\gamma)/\gamma}$, where the compression ratio $r_p = (P_2/P_1)$ and $\gamma = C_P/C_V$.

(iii) Using the result above show that the efficiency of Brayton cycle is

$$\eta = 1 - 1/r_p^{(\gamma-1)/\gamma}.$$

Q.5 (a) With the aid of a diagram give a *brief* description of a Penning gauge. Explain why a Penning gauge must be connected to the vacuum chamber by a high conductance connection.

(b) In a rotary pump, air at $1\ kPa$ enters a displacement volume of 1 *litre* at $20\,°C$ and is swept around with its volume decreasing. Due to the heat of the pump, the air is now at $70\,°C$. During the compression cycle, an air ballast allows $20\ cm^3$ of dry air at $100\ kPa$ and $20\,°C$ to enter. The total volume continues to be compressed until at $133\ kPa$, an exhaust valve opens and air is expelled (at $70\,°C$). What is the volume of gas just prior to expulsion? If the pump turns at $1000\ rpm$, what is the nominal pumping speed? If the pump is connected to a vacuum chamber which contains air and water vapour (28% by volume) at $1\ kPa$, show how the gas ballast operation prevents condensation of water vapour in the pump.

T (°C)	P_{sat} (kPa)
10	1.2276
20	2.339
30	4.246
40	7.384
50	12.349
60	19.940
70	31.19
80	47.39
90	70.14
100	101.33