GX1311

OLLSCOIL NA hÉIREANN
National University of Ireland, Galway

SEMESTER II
SUMMER EXAMINATIONS 1998/99

Second University Examination in Information Technology

LOGICAL FOUNDATIONS OF COMPUTING (CT214)

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Time Allowed : **Three Hours**

Candidates should attempt four questions, two from each section.
Please use separate answer books for each section.
All questions carry equal marks

SECTION A

A1. (a) Describe the NOT, OR and NOR gates, and express each of NOT and OR in terms of NOR.

(b) Construct the logic table for the Boolean expression

$$(x_1 \vee x_2) \wedge \sim(x_1 \wedge x_2)$$

(c) The logic table for a certain Boolean expression is given below:

x_1	x_2	y
0	0	1
0	1	0
1	1	1
1	0	1

Write down an expression for y in disjunctive normal form (DNF). Explain, briefly, why the DNF takes the value 1 precisely when y has value 1.

(d) There are three main symptoms associated with a certain disease. A doctor will judge that a patient has this disease if two of the symptoms are present. Construct a basic machine which models this diagnosis, and explain the notation you use.

A2.(a) A judge hears the following evidence:

- Alan or Bob were responsible
- If Bob is guilty so is Clare
- Clare is innocent

The judge concludes that Alan is guilty. Using a to represent "Alan is guilty", b to represent "Bob is guilty" and c for "Clare is guilty", construct the argument underlying the judges reasoning. Test the validity of the argument.

(b) Define what it means to say that an argument $P, Q \therefore R$ is (i) *valid*, (ii) *sound*. Give an example of a valid argument that is not sound. Show that the argument you gave is valid and explain why it is not sound.

(c) A certain questionnaire has the following two part question:

(i) Give your height in centimetres.

(ii) Describe yourself as either 'tall' or 'short'.*

* **Note:** If your height is 200cm of more you should answer 'tall'.

The researcher studying the responses finds four cases where the answers given to this question are incomplete. The answers given are:

A:- 206cm B:- short C:- tall D:- 159cm

The researcher is, in fact, only interested in seeing if people followed the instructions in the note correctly. Which of the four people, if any, must she ask to complete their answers?

If the instruction had been "Answer 'tall' only if your height is 200cm or over", then who would she have had to contact.

A3.(a) Describe the tableau method for testing a collection of well-formed formulae (compound propositions) for consistency/inconsistency.

(b) Use the tableau method to show that the following collection of well-formed formulae (WFFs) is consistent:

$$p \to q, \quad q \vee r, \quad \sim(p \to r)$$

Read off, from the tableau, an assignment of values for p, q and r which makes all three WFFs take the value 1.

(c) Use the tableau method to show that the following argument is valid:

$$(b \vee c) \to (a \to c), \quad (\overline{b} \to c) \wedge \overline{c} \;\therefore\; \overline{a}$$

SECTION B

B1. (i) Using laws of the propositional calculus, prove the following:

a) $p \wedge \sim(p \wedge q) = p \wedge \sim q$

b) $\sim(p \wedge \sim(p \vee q))$

c) $p \vee (p \wedge q) = p$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

(ii) Two possible definitions of exclusive or are:

$p \text{ XOR } q =_{def} (p \vee q) \wedge \sim(p \wedge q)$

$p \text{ XOR } q =_{def} (p \wedge \sim q) \vee (\sim p \wedge q)$

Prove that these definitions are equivalent.

(iii) I have a program that I wish to test for correctness. I know that if it is correct, then when I run it with an input value of 4, the output should be 10. To test it, I run it with input value 4, and it does indeed return an output of 10. Is there anything wrong with my argument when I say that, using the contrapositive, this shows that the program is correct?

B2. (i) Use the following definitions:

$$(p = q) =_{def} (p \wedge q) \vee (\sim p \wedge \sim q)$$
$$(p \neq q) =_{def} (p \wedge \sim q) \vee (\sim p \wedge q)$$

to prove:

a) $(p \neq q) = \sim(p = q)$

b) $=$ is associative

(ii) Argue that the proposition

$$p_1 = p_2 = \ldots = p_n$$

is false (0) if and only if an odd number of the $p_1, p_2, \ldots p_n$ is false.

(iii) Interpreting b_j to mean "*Bob has sight in j eyes*", comment on the meaning of the proposition

$$b_0 = b_1 = b_2$$

Is this a true or false statement?

B3. (i) Represent the following statements using predicate calculus:

a) In every project team there is at least one systems analyst.

b) Every project team has the same systems analyst.

c) There is no employee who is a member of both team1 and team2.

Assume that *E* is the universe of employees, *PT* is the universe of project teams, and *J* is the universe of job titles. You may use the following atomic predicates:

Isin(t,p) indicates whether employee *p* is part of project team *t*.

EmployedAs(j,p) indicates whether employee *p* has job title *j*.

(ii) Justify the generalisation of de Morgan's law for the existential quantifier:

$\sim(\exists x{:}U \bullet P(x)) = \forall x{:}U \bullet \sim P(x)$

Use this law to prove:

$\sim(\exists x{:}U \bullet R(x) \wedge P(x)) = \forall x{:}U \bullet R(x) \Rightarrow \sim P(x)$

(iii) If no students turn up for an examination is it true to say that all students who took the examination passed? Justify your answer.