

OLLSCOIL NA HÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

FIRST SCIENCE EXAMINATION

MATHEMATICS [MA101]

MA101 — CALCULUS

PASS
First Paper

Professor J. Wiegold
Professor T.C. Hurley
Dr. D. Flannery
Dr. D. O'Regan
Dr. J. Ward

Time allowed: *Three* hours.
Answer six questions.

1. a) Evaluate the following limits

(i) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^3 - x^2 + 3x - 3}$

(ii) $\lim_{x \rightarrow \infty} \frac{x^3 - 5}{8x^3 + x^2 + 1}$

- b) Differentiate the following with respect to x :

(i) $\sin(\sqrt{2x})$ (ii) $(x^{17} - 8x^6 - 4)(2x^9 + 5x)$ (iii) x^{x+1}

2. Let $h(x) = x^3 - 6x^2 + 9x + 2$.

- a) Find the intervals on which h is increasing, and the intervals on which it is decreasing.
- b) Find the intervals on which the graph of h is concave up, and the intervals on which it is concave down.
- c) Find all relative extrema of h and points of inflection of the graph of h .
- d) Sketch the graph of h , labelling the y -axis intercept and the points found in c).

p.t.o.

3. a) Explain (define) the statement “ $f(x)$ is continuous at $x = a$.”

b) Suppose

$$f(x) = \begin{cases} 2x + 5 & x > 2 \\ kx^3 + 1 & x \leq 2 \end{cases}$$

For what value of k is f continuous at $x = 2$?

c) Suppose

$$f(x) = \begin{cases} 4x + 1 & x \leq 1 \\ 2x & x > 1 \end{cases}$$

Find $\lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h}$ and $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h}$

Is $f(x)$ differentiable at $x = 1$? Is $f(x)$ continuous at $x = 1$?

4. State the Mean Value Theorem. Explain the statement “ $f(x)$ is increasing on the interval (a, b) .” Using the Mean Value Theorem, prove that if $f'(x) > 0$ for all $x \in (a, b)$ then $f(x)$ is increasing on (a, b) . By applying the Mean Value Theorem to $2x^4 - 2x^3 - x^2 + x$ on the interval $[0, 1]$, deduce that $8x^3 - 6x^2 - 2x + 1$ has a root in $(0, 1)$.

5. a) Find the equation of the tangent line to the graph of $f(x) = \frac{x-3}{2-x}$ at the point on the graph where $x = 1$.

b) Sketch the region enclosed by the graphs of $y = 2 - x^2$, $y = x$, and the y -axis. Find the volume of the solid generated by revolving this region about the x -axis.

6. a) Use the method of substitution to find

$$\int_0^{\sqrt{\pi}} x \cos x^2 dx.$$

b) Use partial fractions to find

$$\int \frac{5}{x^2 - 3x - 4} dx.$$

c) Use integration by parts to find

$$\int x^2 \ln x dx.$$

d) Use the Fundamental Theorem of Calculus to find

$$\frac{d}{dx} \left(\int_{x^3}^1 \frac{dt}{t^2 + t + 1} \right).$$

p.t.o.

7. a) Use logarithmic differentiation to find $f'(x)$ if

$$f(x) = \frac{\sqrt{x^2 + 9}}{(x^2 + 4)^3 (x^4 + 1)^{100}}$$

- b) Use the partial fraction technique to find

$$\int \frac{(3x^2 - 7x - 3)}{(x - 4)(x^2 + 1)} dx.$$

8. a) Solve the differential equation

$$y' + 2xy = xe^{-x^2}.$$

- b) The population of a certain town at any time t years is given by the solution y of the differential equation

$$\frac{dy}{dt} = ky.$$

In 1991 ($t = 0$), the population of this particular town was 6,000 and in 1997 ($t = 6$) the population had grown to 24,000. In which year will the population reach 96,000?