GX1571

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

FIRST ARTS EXAMINATION

MATHEMATICS [MA120]

MA121 — *CALCULUS*

PASS
First Paper

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Time allowed: *Three* hours.
Answer five questions.

1. (a) Evaluate the following limits:

$$\text{(i)}\ \lim_{x\to 2}\frac{x^3-2x^2+3x-6}{x-2} \qquad \text{(ii)}\ \lim_{x\to 0}\frac{\tan 5x}{\sin 3x} \qquad \text{(iii)}\ \lim_{x\to\infty}\frac{1-2x^3}{x^3+x^2+1}.$$

(b) Differentiate the following functions with respect to x:

$$\text{(i)}\ \frac{x\ln x}{2+\cos x} \qquad \text{(ii)}\ \tan\left(\sqrt{x^4+1}\right).$$

(c) Find the tangent to the curve $y=(x+1)e^{\sin x}$ at the point $(0,1)$.

2. Let $f(x)=x^3-6x^2+16$.

(a) Find the intervals on which f is:
(i) increasing; (ii) decreasing; (iii) concave up; (iv) concave down.

(b) Identify each:
(i) local maximum; (ii) local minimum; (iii) point of inflection of f.

(c) Find the absolute maximum and absolute minimum of f on the closed interval $[-2,3]$, and sketch the graph of f on this interval.

p.t.o.

3. (a) A large snowball is melting at the rate of $3cm^3$ per minute. When the radius is $5cm$, at what rate is it (the radius) changing?

(b) A cylindrical can has its base and lid made of copper costing 2p per cm^2 while its side is made of aluminium costing 1p per cm^2. Find the dimensions of the cheapest such can having volume $32\pi cm^3$.

4. (a) Find the arc length of one turn of the helix given by $h(t) = (3\sin(t), 3\cos(t), 4t), 0 \le t \le 2\pi$.

(b) The area bounded by the graphs $y = 3x, y = x^2$ is revolved about the x-axis. Calculate the volume of the resulting solid.

5. Evaluate three of the following:

(i) $\displaystyle\int \sin^3(\theta)d\theta.$

(ii) $\displaystyle\int \frac{dp}{(4p+1)^4}.$

(iii) $\displaystyle\int x\tan(x^2-1)\ dx.$

(iv) $\displaystyle\int_{-1}^{\infty} x^2e^{-x}\ dx.$

6. (a) Define the natural logarithm function $\ln x$. From the definition, prove that:

$$\ln(\frac{1}{x}) = -\ln x, \qquad x > 0.$$

(b) Find the area between the graphs of the functions $f(x) = x + 2$ and $g(x) = x^2$.

7. (a) Find the general solution to the following differential equations:

(i) $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 9y = 0.$

(ii) $(\cos x)\ \dfrac{dy}{dx} = y\sin x + 5x\cos x.$

(b) A population of bacteria grows according to the law $\dfrac{dN}{dt} = KN$, where $N(t)$ is the number of bacteria at time t (measured in seconds).
If $N(0) = 10^4$ and after two minutes we have a million bacteria, calculate K.