

OLLSCOIL NA HÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

FIRST COMMERCE EXAMINATION

MATHEMATICS [MA130]

MA131

First Paper

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Time allowed: *Three* hours.
Answer six questions.

1. (a) Write the following in the form $(y - a) = -(x - b)^2$:

$$y = -x^2 - 2x + 4$$

Hence sketch roughly the graph of y . (Do not use graph paper, nor make a table of values.)

- (b) Evaluate the following limits:

(i) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 3x - 4}$

(ii) $\lim_{x \rightarrow \infty} \frac{4x^2 - 7x + 2}{2x^2 - 3x - 4}$

(iii) $\lim_{x \rightarrow 0} \frac{\tan 7x}{\sin 3x}$

(iv) $\lim_{x \rightarrow \pi/2} \frac{\cos 3x}{\pi/2 - x}$

p.t.o.

2. (a) Differentiate $\sin x$ from first principles.
- (b) Differentiate the following functions with respect to x

(i) $\sin^4(x^2 + 1)$

(ii) $\frac{(x^2 + 4)\cos x}{2x^2 + 2}$

(iii) $\sqrt{x^2 + \sin x}$

(iv) $e^{\cos x}$

3. (a) A manufacturer has determined a cost function which expresses the annual cost of purchasing, owning, and maintaining its raw material inventory as a function of the size of each order. The cost function is

$$C = \frac{51,200}{q} + 80q + 750,000$$

where q equals the size of each order (in tons) and C equals the annual inventory cost.

- (i) Determine the order size q which minimizes annual inventory cost.
- (ii) What are minimum inventory costs expected to equal?

(b) Let $f(x) = \frac{x^3}{3} - 2.5x^2 + 4x + 2$.

Find $f'(x)$ and the values of x for which $f'(x) = 0$. Find the ranges of values of x for which $f(x)$ is increasing and the ranges for which $f(x)$ is decreasing.

Draw a rough sketch of $y = f(x)$. (Do **not** use graph paper).

Determine the critical points of f and state the nature of these critical points.

p.t.o.

4. (a) The total cost and total revenue functions for a product are

$$C(q) = 500 + 100q + 0.5q^2$$

$$R(q) = 500q$$

- (i) Use the marginal approach to determine the profit-maximising level of output.
- (ii) What is the maximum profit?

- (b) A firm sells each unit of a product for £50. The total cost of producing x (thousand) units is described by the function

$$C(x) = 10 - 2.5x^2 + x^3$$

where $C(x)$ is measured in thousands of pounds.

- (i) Use the marginal approach to determine the profit-maximising level of output.
- (ii) What is the total revenue at this level of output? Total cost? Total profit?

5. (a) Find the indefinite integrals

(i) $\int \left(x^2 + \frac{3}{x^3} - \sqrt{x^5} \right) dx,$ (ii) $\int \frac{4x}{(4x^2 + 5)^3} dx,$

- (b) Use integration by parts and determine

$$\int e^x (x+1)^2 dx.$$

- (c) Use the method of partial functions to determine

$$\int \frac{3x-5}{x^2+2x-15} dx.$$

p.t.o.

6. (a) Evaluate the following definite integrals.

(i) $\int_{-3}^1 \left(\frac{3}{2}x^2 + 4x + 6 \right) dx$ (ii) $\int_1^4 \frac{2x+1}{4x^2+6x} dx.$

- (b) Given $f(x) = 2x^2$ and $g(x) = 27 - x^2$.

- (i) Sketch the two functions.
(ii) For $x \geq 0$ determine the area bounded by the two functions and the y -axis.

7. (a) The rate $w(t)$ at which solid waste is being generated in a major city is described by the function

$$w(t) = 0.5e^{0.025t}$$

where $w(t)$ is stated in millions of tons per year and t equals time measured in years, $t = 0$ corresponding to January 1, 1990.

- (i) Determine the rate at which solid waste is expected to be generated at the beginning of the year 2000.
(ii) What total tonnage is expected to be generated during the 20-year period from 1990 through 2009?

- (b) Find the general and the particular solutions for the differential equations

- (i) $f'(x) = 4x(2x^2 + 3)$, $f(2) = -5$;
(ii) $f''(x) = 3x + 17$, $f'(0) = -10$, $f(2) = 30$.

8. Given the definite integral

$$\int_2^6 (x^2 - 5) dx$$

- (i) Evaluate the integral using explicit rules of integration.
(ii) Approximate the value using the Rectangle Rule, the Trapezoidal Rule, and Simpson's Rule (subdividing into four intervals).
(iii) Determine the most accurate approximation method.