OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

FIRST COMMERCE EXAMINATION

MATHEMATICS [MA130]

MA133

Second Paper

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Time allowed: *Three* hours.
Answer six questions.

1. (a) A firm is developing a TV advertising campaign. Development costs (fixed costs) are £150,000, and the firm must pay £15,000 per minute for television spots. The firm estimates that for each minute of advertising, additional sales of £70,000 result. Of this £70,000, £47,500 is absorbed to cover the variable cost of producing the items, and £15,000 must be used to pay for the minute of advertising. Any remainder is the contribution to fixed cost and profit.

   (i) How many minutes of advertising are necessary to recover the development costs of the advertising campaign?
   (ii) If the firm uses 15 one-minute spots, determine total revenue, total costs (production and advertising), and total profit (or loss) resulting from the campaign.

   (b) A local civic arena is negotiating a contract with a touring riverdance show, *Steps*. *Steps* charges a flat fee of £60,000 per night, plus 40 percent of the gate receipts. The civic arena plans to charge one price for all seats, £12.50 per ticket.

   (i) Determine the number of tickets which must be sold each night in order to break even.
   (ii) If the civic arena has a goal of clearing £15,000 each night, how many tickets must be sold?
   (iii) What would nightly profit equal if average attendance is 7,500 per night?

p.t.o

2. (a) Use **Gaussian Elimination** to give the **complete** solution to:

$$\begin{aligned} x_1 + 2x_2 + 4x_3 - x_4 &= 4 \\ x_1 + 3x_2 - x_3 + x_4 &= 6 \\ x_1 + x_2 - 2x_3 + 2x_4 &= -4 \end{aligned}$$

(b) Use row operations to find the determinant of:

$$\begin{pmatrix} 1 & -2 & 3 & 0 \\ 2 & -5 & -2 & 0 \\ 3 & -5 & 4 & 7 \\ 5 & -3 & -2 & 4 \end{pmatrix}$$

3. (a) Let

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & -3 \\ -3 & 4 & -4 \end{pmatrix}$$

Calculate the adjoint, A^*, of A.
Verify that $AA^* = \alpha I_3$, for some real number α, where I_3 is the identity 3×3 matrix. Find, if possible, the inverse of A.
What is this number α?

(b) Consider the system of equations

$$\begin{aligned} x - y + 2z &= 2 \\ x - 2y - 3z &= -4 \\ -3x + 4y - 4z &= -3 \end{aligned}$$

Rewrite this as

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -3 \end{pmatrix}$$

where A is the matrix as in part (a).
By multiplying through (on the left) by the inverse of A, solve the system of equations.

p.t.o.

4. (a) A firm manufactures two products. Each product must be processed through two departments. Product A requires 2 hours per unit in department 1 and 4 hours per unit in department 2. Product B requires 3 hours per unit in department 1 and 2 hours per unit in department 2. Departments 1 and 2 have, respectively, 60 and 80 hours available each week. Profit margins for the two products are respectively *£3* and *£4* per unit. If x_j equals the number of units produced of product j,

(i) formulate the linear programming model for determining the product mix which maximizes total profit;

(ii) solve using the corner-point method.

What percentage of daily capacity will be utilized in each department?

(b) Find the inverse of

$$A = \begin{pmatrix} 1 & 4 & 6 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

using row operations only. (**DO NOT** use cofactor method).

5. (a) What sum must be deposited today at 8 percent per year, compounded annually if the goal is to have a compound amount of £120,000 twelve years from now?

(b) What is the effective interest rate of an investment at a nominal rate of 7 percent per year, if interest is compounded (i) bimonthly, (ii) quarterly, (iii) continuously?

(c) What is the quotient and the remainder if the polynomial $x^2 + x - 1$ is divided into $x^4 + x - 1$?

6. (a) A mortgage loan of £90,000 is available at an annual interest rate of 6.5 percent. Compute the monthly mortgage payments, the total payment, and the total interest if the loan is for 30 years. What is the difference between the monthly mortgage payments if the loan is for 20 years only?

(b) A family is saving money for a trip to Africa. The trip is planned for three years from now and they want to accumulate £5,000 for the trip. If 12 deposits are made quarterly to an account which earns interest at the rate of 4% per year compounded quarterly, how much should each deposit equal? How much interest will be earned on their deposits?

(c) If 80,000 is to grow to 240,000 over a 12-year period, at what annual interest rate must it be invested, given that interest is compounded semi-annually?

p.t.o.

7. This question requires only the *formulation* of the linear programming model — it does *not* require you to *work out* the optimal solution.

A small refinery blends three petroleum products into two final blends of gasoline. The three components cost the manufacturer £7, £6, and £8 per 100 litres, respectively. The weekly availabilities of the three components are 40,000, 25,000, and 20,000 litres, respectively. The manufacturer sells the two blends at wholesale prices of £14, and £18 per 100 litres respectively. Weekly output should include at least 40,000 litres of blend 1.

The following blending restrictions must be followed.

(a) Component 1 should constitute at least 30 percent of final blend 1 and no more than 20 percent of final blend 2.

(b) Component 2 should constitute exactly 20 percent of final blend 2.

(c) Component 3 should constitute at least 40 percent of final blend 2 and no more than 10 percent of final blend 1.

The objective is to determine the number of litres of each component which should be used in each final blend so as to maximise weekly profit. Formulate the linear programming model for this problem if x_{ij} equals the number of litres of component i used in final blend j.

8. Given the data for a transportation model in the following table. The table lists for origins $i = 1,2,3$ and destinations $j = 1,2,3$ the cost of shipping a unit from origin i to destination j.

Also listed are the supply capacities of the three origins and the demands at each destination.

	Destination			
Origin	1	2	3	Supply
1	30	10	20	300
2	20	20	30	100
3	10	30	10	450
Demand	200	350	300	

(a) Use the northwest corner method to determine an initial solution.

(b) Use the stepping stone algorithm to solve for the optimal solution.