

**OLLSCOIL NA hÉIREANN, GAILLIMH**  
**NATIONAL UNIVERSITY OF IRELAND, GALWAY**

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**SUMMER EXAMINATIONS, 1999**

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**FIRST EVENING COMMERCE EXAMINATION**  
**Enterprise Development and Management**

**MATHEMATICS**

**MA104 --- QUANTITATIVE METHODS FOR BUSINESS**

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Time allowed: *Three* hours  
 Answer five questions.

1. (a) Consider the matrices

$$A = \begin{pmatrix} 2 & 5 \\ 2 & 6 \end{pmatrix} \quad B = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$$

Verify that  $(AB)^{-1} = B^{-1}A^{-1}$ .

- (b) Given the following input-output matrix for two industries X and Y,

	Input to X	Input to Y	Final Demand
Output from X	600	400	200
Output from Y	600	200	0

- (i) Complete the above table using the assumption that total input = total output for each sector.
- (ii) Write down the matrix of technical co-efficients.
- (iii) Calculate the total output required from each industry if the final demands from X and Y change to 500 and 1000 units respectively.

p.t.o.

2. (a) Let  $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 5 & 2 \\ 6 & 3 & 6 \end{pmatrix}$

Calculate:

- (i) the matrix of cofactors of  $A$ ;
- (ii) the adjoint,  $A^*$ , of  $A$ ;
- (iii)  $AA^*$ ;
- (iv) the determinant,  $|A|$ , of  $A$ .

Write down, if possible, the inverse,  $A^{-1}$  of  $A$ .

(b) Consider the system of equations:

$$\begin{aligned} x + y &= 12 \\ 2x + 5y + 2z &= 20 \\ 6x + 3y + 6z &= 0 \end{aligned}$$

Rewrite this as  $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 20 \\ 0 \end{pmatrix}$

where  $A$  is the matrix in part (a) and hence solve the system.

3. A factory makes two models of a product, a standard model and a deluxe model. Each model is started on machine A and finished on machine B. The machine time spent on each model is given in the table below:

Model	Machine A	Machine B
Standard	8 minutes	8 minutes
Deluxe	6 minutes	12 minutes

In a given period machine A can operate for not more than 72 minutes and machine B for not more than 96 minutes. If the profit on a standard model is IR£40 and on a deluxe model is IR£50, how many models of each type should the factory produce in the given period to maximize profits.

4. (a) Differentiate each of the following:

(i)  $y = 3x^2 + \frac{1}{x};$

(ii)  $y = e^x(x^2 - 1);$

(iii)  $y = \frac{4x - 3}{2x + 1};$

(iv)  $y = (1 - \sqrt{x})^6.$

(b) Show, by differentiation, that the function  $f(x) = \frac{1}{x-1}$  has no turning points. Sketch the function indicating the horizontal and vertical asymptotes.

(c) Sketch the quadratic function

$$f(x) = x^2 + 6x + 9$$

making special reference to the maximum/minimum value, x-intercept(s) and y intercept.

5. A firm has a demand function given by  $P(x) = \frac{1}{2}(305 - 6x)$  where  $P(x)$  is the price of the good and  $x$  is the quantity in thousands of units.

The firm also has a cost function

$$C(x) = 300 + 175x - 15x^2 + \frac{x^3}{2}$$

(a) Find the cost and marginal cost for a monthly output of 30,000 units.

(b) If the firm decides to produce with a marginal cost of 175, find the level of output and the cost to the firm.

(c) Find the firm's revenue function,  $R(x)$ .

(d) If the firm produces with a marginal revenue of 32.5, find the output and total revenue.

(e) Find the profit function and hence calculate the profit-maximizing output and maximum profit. Show that when profit is maximized marginal revenue = marginal cost.

(f) Draw a rough sketch of the profit function.

6. (a) £5,500 is deposited at a nominal rate of 6%. Calculate the value of the investment in 3 years time if interest is compounded.
- (i) Every 3 months;
  - (ii) Monthly;
  - (iii) Daily;
  - (iv) Continuously;
- (b) Calculate the present value of £20,000 that is expected to be received in 6 years time with an interest rate of 12% compounded annually.
- (c) A person wants to deposit £5000 per year for 5 years. If interest is earned at the rate of 10% per year, compute the amount to which the deposit will grow by the end of the 5 years if:
- (i) Deposits of £5000 are made at the end of each year with interest compounded annually;
  - (ii) Deposits of £2500 are made at the end of each 6 month period with interest compounded semi-annually.
  - (iii) Deposits of £1250 are made at the end of every quarter with interest compounded quarterly.

7. The frequency distribution given below summarises the sale prices (in thousand £) of 300 houses:

Sale price [£000]	Frequency
35-45	33
45-55	133
55-65	71
65-75	44
75-85	17
85-95	2

- (i) Calculate the mean and standard deviation for the data.
- (ii) Estimate the median sale price.
- (iii) Given that the upper and lower quartiles are 63.31 and 48.15 respectively and that the two highest data values are 85.2 and 86.3, and the two lowest are 35.6 and 37.9, construct a boxplot to represent the data.

p.t.o.

8. (a) A student representative committee of 3 persons is to be selected from a group of 6 men and 4 women. If the members of the committee are selected at random from the group, what are the probabilities that:
- (i) 3 women are selected;
  - (ii) 3 men are selected;
  - (iii) the committee is made up of a mixture of men and women?
- (b) (i) In an industrial plant, machines A, B and C account respectively for 70%, 20% and 10% of total output. 3% of units produced by machine A are defective and the corresponding percentages for machines B and C are 4% and 5%, respectively. If a unit is selected at random from a large stockpile, what is the probability that it will be defective?
- (ii) Given that the unit selected is defective, what is the probability that it was manufactured by machine A?