

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

SECOND UNIVERSITY (Arts and Science) EXAMINATION

MATHEMATICS [MA212]

PASS

Professor J. Wiegold,
Professor T.C. Hurley,
Dr. A. Christofides,

Time allowed: *Two* hours.
Full marks for *three* questions.

1. (a) Let $w = (x^2 + y^2)^{-1}$. Prove that

$$x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} + 2w = 0.$$

- (b) Let

$$w = \frac{x - 2y}{3x + y}.$$

Show that

$$\frac{\partial^2 w}{\partial x^2} - 6 \frac{\partial^2 w}{\partial x \partial y} + 9 \frac{\partial^2 w}{\partial y^2} = 0.$$

2. Let S be the surface

$$z = x^3 - 3x^2y + 5xy^2 - x + y.$$

Let M and N be the tangent planes to S at the origin $O(0,0,0)$, and at the point $Q(1,-1,7)$, respectively. Find the equations of the planes M and N and the parametric equations of the normal line to S at O . Let L be this line. Find the point of intersection of L with the plane N , and the distance of this point from the plane M .

p.t.o.

3. (a) Let

$$f(x, y) = 3x^2y - xy.$$

Find the directional derivative of $f(x, y)$ at the point $P(2, 3)$, in the direction $(1, 1)$. Find the direction of steepest slope and the direction of constant altitude at the point P .

- (b) Find the stationary points of the function

$$z = 2xy^2 - x^2y + 4xy$$

and determine if they are maxima, minima or saddle points.

4. (a) Let $z = e^{xy}$, $x = 2u + v$, $y = u/v$. Use the chain rule to evaluate the derivatives

$$\frac{\partial z}{\partial u} \quad \text{and} \quad \frac{\partial z}{\partial v}.$$

- (b) Find the maximum and minimum values of $x^2 + y^2$, subject to the condition

$$5x^2 + 8xy + 11y^2 = 39.$$