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THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

SECOND UNIVERSITY EXAMINATION

MATHEMATICS [MA200]

MA203 - LINEAR ALGEBRA

PASS

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Time allowed: *Two* hours.

Attempt *three* questions.

1. (a) Find the general solution of the following system of linear equations:

$$\begin{aligned} x - 2y + 3z - 2t &= 1 \\ 3x + 4y - z - 6t &= 13 \\ x - y + z + 4t &= 5 \end{aligned}$$

(b) Find the values of k such that the following system has (i) a unique solution; (ii) no solution and (iii) infinitely many solutions:

$$\begin{array}{cccl} x & -2y & +2z & = 1 \\ x & & -2kz & = 5 \\ 2x & +ky & +4z & = 8 \end{array}$$

(c) Evaluate $\begin{vmatrix} 1 & -2 & 3 & 1 \\ 4 & -4 & 4 & 4 \\ 3 & -8 & 4 & 2 \\ 0 & -1 & 3 & 1 \end{vmatrix}$.

2. (a) Find a Cartesian equation of the plane which passes through the point $(3, 1, -2)$ and is normal to the vector $-\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$.

(b) Find a parametric equation of the line of intersection between the two planes $x - y + 2z = 2$ and $2x - 3y + z = 1$.

(c) Find a Cartesian equation of the line which passes through the point $(3, 1, -2)$ and is normal to the plane $4x - y + z = 2$.

(d) Use the Gauss-Jordan elimination method to show that

$$\begin{pmatrix} 1 & 2 & -1 \\ -1 & -1 & 3 \\ 2 & 5 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} 14 & 3 & -5 \\ -5 & -1 & 2 \\ 3 & 1 & -1 \end{pmatrix}$$

p.t.o.

3. In a certain town a total of 40,000 journeys are made into the town every week. These journeys are made by people using either their own transport, buses or trains. Each week 10% of those who had used their own transport the previous week switch to travelling by bus and another 10% switch to using a train. Similarly, 10% of the bus users switch to using their own transport, and another 20% start using the train. And 30% of train users turn to private transport and another 10% switch to using the bus.

(a) Show that one transition matrix for this process is $A = \begin{pmatrix} 0.8 & 0.1 & 0.3 \\ 0.1 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.6 \end{pmatrix}$

(b) If the process is in its steady state, how many journeys into the town will be made by bus every week?

(c) Show that eventually the system will tend to its steady state, regardless of the initial distribution of journeys.

Hint It may save you time to note that $\begin{pmatrix} 2 & -1 & -2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 \\ 0 & -4 & 4 \\ -1 & 3 & -1 \end{pmatrix}$.

4. (a) Find an orthogonal matrix E and a diagonal matrix D such that $A = EDE^{-1}$ where:

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Find E^{-1}.

(b) It is known that the matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ has eigenvalues 2, 5 and 3 and corresponding to these values it has eigenvectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, respectively. Use this information to describe the surface

$$3x^2 + 4y^2 + 3z^2 - 2xy - 2yz = 1$$

END OF PAPER