GX1584

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 1999 — PASS

THIRD ARTS and SCIENCE EXAMINATIONS

MATHEMATICS and APPLIED MATHEMATICAL SCIENCE

MA314 — Algebra

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Time allowed: *two* hours.
Full marks for *three* questions.

1. (i) Use the Gram–Schmidt process to find an orthogonal basis for the subspace U of $\mathbf{R}^4$ consisting of the solutions of the equation
$$3x_1 - 2x_2 + x_3 - 3x_4 = 0.$$
(ii) By completing the square, or otherwise, express the quadratic form
$$2x^2 + 3y^2 + z^2 - 4xy + 4xz$$
as a linear combination of squares of homogeneous linear polynomials.
(iii) Suppose that $\mathbf{u}_1$, $\mathbf{u}_2$ are orthogonal vectors. Prove that if $\mathbf{v}$ is any vector, and if
$$\mathbf{p} = \frac{\mathbf{v}\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{v}\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2$$
then $\mathbf{p}\cdot\mathbf{u}_i = \mathbf{v}\cdot\mathbf{u}_i$ $(i = 1, 2)$.

2. By applying the Gram–Schmidt process to the functions 1, x, x^2, find polynomials $u_0(x)$, $u_1(x)$, $u_2(x)$ which are orthogonal with respect to the inner product
$$f\cdot g = \int_0^1 f(x)g(x)\,dx.$$
Then use $u_0(x)$, $u_1(x)$ to find a polynomial of degree 1 (or less) which approximates the function
$$f(x) = x^3.$$

3. Let
$$A = \begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$
Find an orthogonal matrix P such that P^tAP is diagonal.
Sketch the surface $2x^2 - y^2 + 2z^2 - 4xy + 2xz + 4yz = 1$ with respect to suitable axes along eigenvectors of A.

4. Consider he problem of minimising the expression $9y_1 + 3y_2 + 5y_3$ subject to the constraints
$$\begin{aligned} y_1 + 2y_2 - 2y_3 &\geq 6 \\ y_1 - 2y_2 + 3y_3 &\geq 2 \\ y_1,\, y_2,\, y_3 &\geq 0 \end{aligned}$$
Formulate the dual problem, and solve both problems.