GX1590

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

B.Sc. (Part I) and B.A. DEGREE EXAMINATION

MATHEMATICS
[MA342 - *TOPOLOGY]*

HONOURS

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Time allowed: *Two* hours
Answer three questions

1. Let X be a topological space.
   (a) Let Y be a subspace of X, A a subset of Y and $A = B \cap Y$ with B closed in X. Show A is closed in Y.

   (b) Let C and D be subsets of X. Show
   $$\overline{C \cap D} \subseteq \overline{C} \cap \overline{D}.$$

   (c) Let X be a Hausdorff space, E a subset of X and $x \in X$ a limit point of E. Show every neighborhood of x contains infinitely many points of E.

   (d) Let X be a Hausdorff space. Show
   $$F = (X \text{ x } X) \setminus \{(x,x) : x \in X\}$$
   is open in $X \text{ x } X$.

p.t.o.

2. (a) Let X and Y be topological spaces. Show the following are equivalent:

   (i) $f : X \to Y$ is continuous;

   (ii) for every subset A of X, $f(\bar{A}) \subseteq \overline{f(A)}$;

   (iii) for every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

   (b) Let X and Y be topological spaces with X metrizable and $f : X \to Y$. Suppose for every sequence x_n converging to x in X the sequence $f(x_n)$ converges to $f(x)$. Show $f : X \to Y$ is continuous.

   (c) Let X and Y be topological spaces and $f : X \to Y$ continuous. Show for every $B \subseteq Y$ that

$$\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B}).$$

   (d) Consider the family of topological spaces $\{X_\alpha : \alpha \in J\}$. Let $X = \prod_{\alpha \in J} X_\alpha$ have the product topology, let A be a topological space and let $f : A \to X$ be given by $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each a. Suppose $f : A \to X$ is continuous. Show $f_\alpha : A \to X_\alpha$ is continuous for each $\alpha \in J$.

3. (a) Show that the product of two connected spaces is connected.

   (b) Let X be a topological space. Show that the components of X are connected.

   (c) Suppose the topological space X is disconnected. Show there exists a continuous mapping of X onto the discrete space $\{0,1\}$.

   (d) Let X be a topological space and let $\{A_\alpha\}$ be a collection of path connected subsets of X with $\bigcap A_\alpha \neq \phi$. Show $\bigcup A_\alpha$ is path connected.

4. (a) Show that every compact subset of a Hausdorff space is closed.

   (b) Show that the continuous image of a compact space is compact.

   (c) Suppose X is a sequentially compact space. Show X is limit point compact.

   (d) Define the term *net* in a topological space. Let A be a subset of a topological space X with $y \in \overline{A}$. Show there exists a net consisting of elements of A converging to y.