

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

FIRST YEAR EXAMINATIONS IN FINANCIAL MATHEMATICS AND ECONOMICS

MA111 - MATHEMATICS OF FINANCE

Time allowed: *Three* hours.

Full marks for *seven* questions.

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Dr. A. Christofides

1. Explain the following concepts relating to an investment:

- (a) The accumulation function $a(t)$.
- (b) The effective rate of interest for the n -th period.
- (c) The effective rate of discount for the n -th period.

Show that the effective rate of interest for the n -th period, of an investment earning simple interest at the rate of i , is a decreasing function of n , while the effective rate of interest for the n -th period, of an investment earning compound interest at nominal rate of i , payable m times per period, is constant.

2. At what rate of simple interest will £2000 accumulate to £2600 in two and a half years? What sum of money should be deposited, at the same rate of simple interest as the previous sum, in order to accumulate to £3000 in three years? What is the effective rate of discount of this second investment in the third year?
3. (a) A sum of £5000 is invested at a certain effective rate of interest for two years and earns £710 of interest. What is the present value of an investment which will yield £3000 in three years time invested at the same rate of interest?
- (b) At a certain effective rate of interest, the present values of the following two investments are equal:
- (i) £2000, invested at the end of 5 years, plus £1500, invested at the end of 10 years.
 - (ii) £3000, invested at the end of 5 years.

At the same rate of interest, £1000, invested now, plus £1200, invested at the end of 5 years, will accumulate to P at the end of 10 years. Calculate P .

4. What nominal rate, payable quarterly, is equivalent to, (a) an effective rate of 6% per annum, (b) a nominal rate of 7% payable semiannually, (c) a nominal rate of 7% compounded continuously.

Find the accumulated amount of an investment of £10,000 invested for 3 years at 10 %, compounded continuously.

p.t.o.

5. The method of equated time gives an approximate value for the moment in time at which a single payment is equivalent to several payments, made at various moments in time. Give a brief description of this method and explain why the answer obtained with it is always greater than the exact answer.

Payments of £1500, £3000, and £4000 are due at the end of two years, five years and eight years, respectively. The effective rate of interest is 5% per annum. Use the method of equated time to find an approximate value for the moment in time at which a single payment of £8500 would be equivalent to the three previous payments. Find the exact answer.

6. (a) Describe the "rule of seventy two" for determining the length of time it takes for an investment to double in value and explain why it works.

- (b) A loan of £9000 is due in three years time and a second loan of £13,000 is due in eight years time. If these debts were to be discharged by a payment of £4000 in four years time and a further payment at the end of six years, what would be the size of the second payment, if the rate of interest is 8%, compounded quarterly?

7. Find the present value of an annuity which pays £400 at the end of each month for 20 years, and of one with the same number of payments, which pays £400 at the beginning of each month. Find the value of both annuities at the end of the 12th year. Find the present value of the perpetuity which pays £400 at the end of each month. The rate of interest, in each case, is 7.2% convertible monthly.

8. The present value of an annuity of £2000, paid at the end of every quarter for five years, is equal to £18,000. Determine the rate of interest i of the annuity, using tables and linear interpolation. Explain the approximation

$$i \simeq \frac{1 - \left(\frac{9}{20}\right)^2}{9}.$$

9. A loan is repaid with 12 payments of £1000 followed by 8 payments of £800 at the end of each half-year. The nominal rate of interest is 8% convertible semi-annually. Find the outstanding loan balance immediately after eight payments have been made, both by the prospective and by the retrospective method.

10. (a) A loan of £3500 is being repaid by payments of £200 at the end of each quarter for as long as necessary, plus a smaller final payment. The nominal rate of interest convertible quarterly is 8%. Find the amount of the principal and interest in the fourth payment.

- (b) A has borrowed £10,000, at a rate of 10% effective. Exactly one year later, A started accumulating a sinking fund at 8% effective, to repay the loan. At the end of each year A makes a total repayment of £1500.

- (i) How much of the £1500 pays the interest on the loan?
- (ii) How much of the £1500 goes into the sinking fund?
- (iii) What is the balance in the sinking fund at the end of five years?
- (iv) How much of the sixth payment should be considered as principal and how much should be considered as interest?