GX1596

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SUMMER EXAMINATIONS 1999

SECOND UNIVERSITY EXAMINATION

MATHEMATICS [MA283]

HONOURS
Second Paper

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Time allowed: *Two* hours.
Answer *three* questions.

A1. Explain the terms (i) spanning set and (ii) linearly independent set as applied to a set of vectors $v_1, v_2, \cdots, v_n$ of a finite-dimensional vector space V.
Let $u_1, u_2, ..., u_m$ be a linearly independent set and $v_1, v_2, ..., v_n$ be a spanning set in V. Prove that $m \leq n$.
Show that the vectors:

$$\begin{aligned} v_1 &= (1,-1,2,1,3), & v_2 &= (2,1,0,1,1) \\ v_3 &= (-1,2,1,-1,0), & v_4 &= (3,-4,1,-1,-2) \end{aligned}$$

are linearly independent in R^5.

A2. Let f be a linear mapping from a vector space U to a vector space V. Prove that $Kerf = \{u \in U : f(u) = 0_v\}$ and $Imf = \{v \in V : v = f(u) \text{ for some } u \in U\}$ are subspaces of U and V respectively.
Prove that

$$\dim U = \dim Kerf + \dim Imf.$$

For the linear transformation defined by the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & -1 & 1 \\ 1 & 3 & 1 & 1 & -1 \\ 0 & 1 & 1 & 2 & -2 \\ 3 & 6 & 0 & 0 & -6 \\ 1 & 5 & 3 & 5 & -5 \end{pmatrix}$$

find a basis for ImA.

p.t.o.

A3. Let L be the linear mapping $L : R^4 \to R^3$ given by

$$L(x, y, z, w) = (x + 2y + w,\ \ x + 3y + z + 2w,\ \ 2x + 5y + z + 3w).$$

Find the natural matrix of L.
Find the matrix of L with respect to the bases:

$$\left\{\begin{array}{lcl} v_1 & = & (1,0,0,0) \\ v_2 & = & (0,1,0,0) \\ v_3 & = & (-2,1,-1,0) \\ v_4 & = & (1,0,1,-1) \end{array}\right. \quad \text{for } R^4 \text{ and} \quad \left\{\begin{array}{lcl} w_1 & = & (1,1,2) \\ w_2 & = & (2,3,5) \\ w_3 & = & (0,0,1) \end{array}\right. \quad \text{for } R^3$$

A4. Let A be an $n \times n$ matrix. Prove that A is diagonalisable if the eigenvectors of A form a basis.
Let A be symmetric with real number entries.
Prove that the eigenvalues of A are real.

$$\text{Let } A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

Find an orthogonal matrix O and a diagonal matrix D such that

$$A = ODO^T.$$