GX1597

OLLSCOIL NA ÉIREANN GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 1999

MATHEMATICS

MA414 --- *AUTOMATED REASONING*

Professor J. Wiegold
Professor T. Hurley
Dr. M. McGettrick

Time allowed: *Two* hours.
Attempt three questions.

1. (i) Define the inference rule *Hyperresolution*. Using Hyperresolution, write down the new clause generated (if any) in each of the following:

(a) `-P(x)|Q(z).`   `P(a)|S(b).`
(b) `-P(x,y)|Q(x)|-R(y).`   `P(a,b).`   `R(z)|S(z).`
(c) `-Q(x,y,z)|-P(z)|-S(z,y).`   `P(x).`   `S(a,b).`

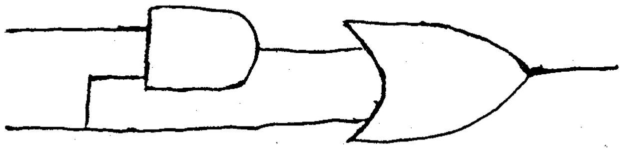
(ii) Explain what is meant by a *skolem constant* and a *skolem function*. Hence, rewrite the following, eliminating existential quantifiers:

(a) $(\exists x)A(x)$
(b) $(\forall x)(\exists y)(\forall z)A(x,y,z)$
(c) $(\forall x)(\forall y)(\exists z)B(z,z)$

(iii) Prove that any truth function $f:\Omega^n \to \Omega,\ \Omega = \{T,F\}$, can be written using the NAND function.

p.t.o.

2. (i) Using demodulation, write an OTTER program to re-write the logic circuit



completely in terms of NAND gates.

(ii) Let G be a group such that the square of any element of G gives the identity. Write an OTTER program to prove that G is abelian.

(iii) Explain what is meant by saying a statement is in *Conjunctive Normal Form.*

3. Let L denote our axiomatization of Propositional Calculus.

(i) Prove that every theorem in L is a tautology.

(ii) State and prove the deduction theorem for L.

(iii) What is meant by saying L is (a) complete, (b) consistent?

(iv) Prove that $A \to (B \to C), B \vdash_L A \to C$

4. (i) Let K be a first order theory. What is meant by (a) a proper axiom of K, (b) a theorem of K, (c) an interpretation of K?

(ii) Write down the Axiom Schema for First Order Predicate Calculus. From these and the inference rules of Modus Ponens and Generalization, prove the theorem

$$\vdash_K (\forall x)(\forall y) A(x,y) \to (\forall y)(\forall x) A(x,y)$$

(iii) Describe a first order theory suitable for abelian Groups.