OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

THIRD ARTS, ENGINEERING & SCIENCE EXAMINATIONS

APPLIED MATHEMATICAL SCIENCE [AS300]

MM355 — *NUMERICAL ANALYSIS*

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Time allowed: *Three* hours.
Those answering both sections should attempt *five* questions, but no more than *three* from each section.
Those answering one section should attempt *three questions.*
PLEASE USE SEPARATE ANSWER BOOKS FOR EACH SECTION.

SECTION A

1. (a) Consider the sequence $\{x^k\}_{k=0}^{\infty}$ defined by $x^k = Tx^{(k-1)} + c,\ k \in \{1, 2, ...\}$ and $c \neq 0$ with T an $n \times n$ matrix.
If $\| T \| < 1$, explain why the sequence $(x^{(k)})$ converges for any $x^{(0)} \in \mathbf{R}^n$ to a vector $x \in \mathbf{R}^n$.
Derive the error bound
$$\| x - x^{(k)} \| \leq \frac{\| T \|^k}{1 - \| T \|} \ \| x^{(1)} - x^{(0)} \| \, .$$

(b) Consider the linear system
$$\begin{array}{rcrcrcr} 4x_1 & + & x_2 & + & x_3 & = & 6 \\ x_1 & + & 5x_2 & + & 2x_3 & = & -14 \\ x_1 & + & 2x_2 & + & 4x_3 & = & 2 \end{array}$$

(i) Taking $x^{(0)} = (0, 0, 0)^t$ perform *two* iterations of the Jacobi method.
(ii) Find the matrix T and the vector c such that the Jacobi method for solving these equations can be written in the form $x^{(k)} = Tx^{(k-1)} + c$.
(iii) Taking $x^{(0)} = (0, 0, 0)^t$ perform *one* iteration of the SOR method with $w = 1.25$.

p.t.o

2. (a) Let

$$A = \begin{pmatrix} 1/3 & 1/6 \\ 1/6 & 1/2 \end{pmatrix}.$$

Find $||A||_2$. Why is A convergent?

(b) Let A be a real $n \times n$ matrix with

$$R_i = \left\{ z \in \mathbf{C} : |z - a_{ii}| \leq \sum_{\substack{j=1 \\ j \neq i}}^{n} | a_{ij} | \right\}.$$

Show the eigenvalues of A are contained in $\bigcup_{i=1}^{n} R_i$.

(c) Use part (b) together with properties of Sturm sequences to separate the eigenvalues of the tridiagonal matrix.

$$A = \begin{bmatrix} 5 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

i.e. find disjoint intervals each of which contains *one* eigenvalue.

3. (a) For the matrix

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 1 & 1 \\ 2 & -6 & 1 \end{pmatrix},$$

find an orthogonal matrix K such that KA is upper triangular. Now write down the QR factorization of A.

(b) For the symmetric matrix

$$B = \begin{pmatrix} 5 & 3 & 4 \\ 3 & 1 & 1 \\ 4 & 1 & 3 \end{pmatrix},$$

use Householder's method to compute a tridiagonal matrix with the same eigenvalues as B.

p.t.o.

4. (a) The fourth order Adams method applied to $y' = f(t, y)$ uses the formulae

$$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$$

as predictor and

$$y_{n+1} = y_n + \frac{h}{24}(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2})$$

as corrector.
Consider the initial value problem

$$\begin{cases} y' = -y + 2, & 0 \leq t \leq 1 \\ y(0) = 0 \end{cases}$$

with $h = 0.1$. Assuming that $y(0.1) = \cdot 19032, y(0.2) = \cdot 36254$ and $y(0.3) = \cdot 51836$ use the fourth order Adams method to estimate $y(0.4)$

(b) Show that the finite difference method for the boundary value problem

$$\begin{cases} y'' & = & 40y' + 4y + 64t, \quad 0 \leq t \leq 1 \\ y(0) & = & 3, y(1) = 4 \end{cases}$$

with $N = 3$ reduces to solving

$$\begin{pmatrix} -9/4 & 4 & 0 \\ -6 & 9/4 & 4 \\ 0 & -6 & 9/4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 17 \\ -2 \\ -19 \end{pmatrix}.$$