

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 1999

COMPUTER ALGEBRA [CS407]

Professor J. Wiegold

Professor T. Hurley

Dr. G. Pfeiffer

Time allowed: *Two* hours.

Attempt *all three* of the following problems.

Problem 1. Integer Arithmetic.

- Explain how to write an integer in some base b . Write the number 456123 in bases 2, 10 and 16.
- Choose a base b and design a data structure to represent positive integers in that base in GAP or C.
- Write a GAP (or C) program that determines the sum of two positive integers represented in your base b .

Problem 2. Prime Numbers.

- Name **three** different strategies to find primes and describe **two** of them in more detail.
- Recall the definition of a pseudo prime and state the theorem which relates these numbers to primality tests.
- Discuss the relation between prime numbers and computer algebra.

Problem 3. Group Theory.

- Either**, give a list of GAP commands that can be used to determine the order of the group generated by the permutations

$$a = (1, 12, 6, 8, 9, 2)(3, 7, 4, 10, 11, 5) \quad \text{and} \quad b = (1, 4, 5, 3, 11)(2, 12, 7, 6, 9)$$

or, give a list of GAP commands that can be used to determine the order of the group given by the presentation

$$G = \langle a, b \mid a^2 = b^3 = (ab)^5 = 1 \rangle.$$

- Describe, in your own words, the Todd-Coxeter procedure. What is its input data? what is the output? Explain the strategy that is used to determine the output. Why is this called a "procedure" rather than an "algorithm"?
- Define the concept of a group action. What is the orbit and the stabilizer of a point? How do these two help determine the size of a permutation group?