GX1677

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NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2000

FIRST YEAR
MATHEMATICAL PHYSICS (MP102)
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Paper 2
Time allowed: *THREE* hours.
Attempt *SIX* questions.

1. Two cars A and B are initially at rest side by side. A starts off on a straight track with acceleration of 2 m/s^2 . Five seconds later B starts off on a parallel track to A with acceleration of 3.125 m/s^2.

   (a) Calculate the distance travelled by A after 5 seconds.

   (b) Calculate the time taken by B to catch up with A.

   (c) Find the velocities of A, B at that time.

   (d) Suppose that B continues at a constant **velocity** after this time. How long does it take A to catch up with B again?

2. A block A of mass $4m$ lies on a rough inclined plane of angle 30^o with coefficient of friction μ. It is connected via a light inextensible string, passing over a fixed pulley at the top of the plane, to a vertically hanging block B of mass m.

   (a) Show that the block moves down the plane provided $\mu < \frac{1}{2\sqrt{3}}$.

   (b) Find the acceleration when $\mu = \frac{1}{4\sqrt{3}}$ and determine the distance moved in 5 secs.

3. A particle is projected upwards from a point O at angle of projection of 45^0. It passes through a point P whose horizontal and vertical displacements from O are $3L$ and L respectively.

   (a) Find the initial velocity of the particle.

   (b) Find the velocity vector of the particle when it passes through P.

   (c) How far does the particle travel horizontally before hitting the ground again?

4. The combined mass of a man and his bicycle is 100 kg. He cycles up a hill of height 20 meters with incline 1:10 at a velocity of 5 m/s.

   (a) If his power output is 500W, find the frictional force acting assuming that it is constant.

   (b) He then freewheels back down the hill from being initially at rest. Use the Principle of Work to find his final velocity on reaching the bottom of the hill.

5. A car travels around a corner in a circular path of radius 5 m at a constant speed. It is observed that if the car's speed is 12 km/hr, then it just begins to skid.

   (a) Assuming the same frictional force is acting, calculate the car's smallest possible turning radius if the speed is 30 km/hr.

   (b) Calculate the turning radius for the car travelling at 12 km/hr in wet conditions where the frictional force is halved.

   (c) Calculate the minimal stopping distance for a car travelling along a line at 12 km/hr in wet conditions.

6. A bungee jumper of mass 80 kg stands on a high bridge and is tied to a 5 m elastic rope with a Young's modulus of 400 N. You may assume that $g \simeq 10\ \mathrm{ms}^{-2}$.

   (a) Assuming that the other end of the rope is tied to the bridge, calculate how far he falls before coming to rest.

   (b) Find the time taken to come to rest.

7. (a) Find the solution to differential equation

$$\frac{d^2x}{dt^2} + 4x = \cos t$$

   subject to the conditions $x(0) = dx/dt(0) = 0$.

   (b) Find the solution to the equation

$$\frac{d^2x}{dt^2} + 4x = \cos 2t$$

   subject to the conditions $x(0) = dx/dt(0) = 0$, and discuss the behaviour of the solution for large values of t.

8. A particle of mass m is attached to one end of a light elastic string of modulus mg and natural length a. The other end is attached to a fixed point O on a smooth horizontal table. The particle is placed on the table and the string is stretched to a length $3a$ when the particle is projected perpendicular to the string with speed u. In the subsequent motion, the maximum length of the string is $5a$. Find u^2.

9. A uniform rod of mass $6m$ and length $2a$ can freely rotate about a horizontal axis at one end and carries a particle of mass $2m$ at the other end. The rod is released from rest when horizontal. Find its angular speed when it makes an angle θ with the vertical and find the components along and perpendicular to the rod, of the reaction at its axis.

10. A uniform ladder, of weight W, rests with one end against a smooth vertical wall and the other on a smooth horizontal floor. Its lower end is attached by means of a light inextensible string to the junction of the wall and the floor. A man whose weight is W climbs the ladder. Show that as the man moves from one-sixth way up the ladder to five-sixths up the ladder, that the tension in the string doubles.