GX1679

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2000

Second Information Technology

Methods of Mathematical Physics (MP201)

Professor M.J. Sewell;
Professor J.N. Flavin;
Dr. M.S. Ó Confhaola.

Time allowed: *Two* hours
Full marks for THREE questions

1. The function $f(x)$ is defined as

$$f(x) = \begin{cases} -1 & ,-1 \le x < 0 \\ 2x & ,0 \le x < 1 \end{cases}$$

and elsewhere by periodicity, period = 2.

(a) Sketch the function in the interval $[-3, 3]$

(b) Find its Fourier Series.

(c) State the the value to which the series converges to at $x = 0$. Hence prove that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots\dots\dots$$

2. (a) Show that if

$$f(x,y) = \frac{xy}{x+y}, \text{ then } x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy} = 0$$

(b)Let

$$f(x,y) = \frac{x+y}{x-y} \text{ where } x = \cos t \text{ and } y = \sin t$$

Use the Chain Rule $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$ to find $\frac{df}{dt}$ and check your result by direct differentiation.

3. (a) Find and classify the stationary points of the function

$$f(x,y) = x^3 + 3y^3 - \frac{1}{2}x^2 - 2x - 9y$$

(b) Use the method of Lagrange Multipliers to find the stationary points of the function

$$f(x,y,z) = x^2 + y^2 + z^2$$

subject to the constraint $3x + y + 2z = 4$

4. (a) Find the rate of change of the function

$$f(x,y,z) = xz^3 + x^2y^2$$

at the point P$(-2,1,3)$ in the direction of the vector $6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$

(b) Show that the point $(2,-1,2)$ lies on each of the surfaces

$$x^2 + y^2 + z^2 = 9 \text{ and } x^2 + y^2 - z = 3$$

Find a unit normal to each surface at this point and deduce the angle between the surfaces at this point.

5. (a)Sketch the region A enclosed by the line $y = x$ and the parabola $y = x^2$

Find the area of the region by evaluating the integral

$$\iint dydx$$

(b)Consider the integral

$$\int_0^1 \int_x^1 e^{y^2} dydx$$

Sketch the region of integration. Change the order of integration and evaluate the integral

(c) Using plane polar coordinates evaluate the integral

$$\iint_R e^{-(x^2+y^2)} dxdy$$

where R is the region enclosed by the circle $x^2 + y^2 = 1$

6. (a) Evaluate the integral

$$\int_0^1 \int_0^z \int_0^{\sqrt{1-y^2}} xdxdydz$$

(b) Find the volume V enclosed by the planes $x = 0, y = 0, z = 0$ and the plane $x + 2y + 3z = 1$ by evaluating

$$\iiint_V dxdydz$$