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OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2000

Second Science and Arts Examination

Methods of Mathematical Physics (MP201)

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Time allowed: *Two* hours
Full marks for THREE questions

1. (a) Find the rate of change of the function

$$f(x,y,z) = xz^3 + x^2y^2$$

at the point P(–2, 1, 3) in the direction of the vector $6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$

(b) Show that the point $(2, -1, 2)$ lies on each of the surfaces

$$x^2 + y^2 + z^2 = 9 \quad \text{and} \quad x^2 + y^2 - z = 3$$

Find a unit normal to each surface at this point and deduce the angle between the surfaces at this point.

2. (a)Sketch the region A enclosed by the line $y = x$ and the parabola $y = x^2$

Find the area of the region by evaluating the integral

$$\iint dydx$$

(b)Consider the integral

$$\int_0^1 \int_x^1 e^{y^2} dydx$$

Sketch the region of integration. Change the order of integration and evaluate the integral

(c) Using plane polar coordinates evaluate the integral

$$\iint_R e^{-(x^2+y^2)} dxdy$$

where R is the region enclosed by the circle $x^2 + y^2 = 1$

3. Evaluate the line integral

$$\int_{(0,0,0)}^{(4,2,-3)} \mathbf{V} \bullet d\mathbf{r}$$

for the vector field $\mathbf{V} = 2yz\mathbf{i} - (x^2 + y^2)\mathbf{j} + 5xz\mathbf{k}$ along each of the following paths:

the successive straight line segments from $(0,0,0)$ to $(0,2,0)$ to $(4,2,0)$ to $(4,2,-3)$

the curved line $x = 4t^3,\ y = 2t^2,\ z = -3t$ from $t = 0$ to $t = 1$

the straight line joining $(0,0,0)$ directly to $(4,2,-3)$: $x = 2y,\ z = -\frac{3}{2}y$

Can you deduce, based on your answers, whether the vector field $\mathbf{V}$ is conservative or not? Justify your answer.

4. The divergence theorem of Gauss can be written in the form

$$\iiint_V \nabla \bullet \mathbf{A}\,dV = \iint_S \mathbf{n} \bullet \mathbf{A}\,dS$$

where V is a volume, S is the closed bounding surface of V and $\mathbf{n}$ is the outward unit normal vector on S.

Verify this result for the vector field

$$\mathbf{A} = 2xz\mathbf{i} + yz\mathbf{j} + 3yz\mathbf{k}$$

if V is the finite volume bounded by the planes $x = 0,\ y = 0,\ z = 0,\ x = 2,\ y = 4,\ z = 3.$

5. Find all the solutions of the equation

$$\frac{\partial}{\partial t}u(x,t) = 5\frac{\partial^2}{\partial x^2}u(x,t)\,, \qquad 0 < x < 15,\ t > 0$$

subject to the boundary conditions

$$u(0,t) = \frac{\partial u}{\partial x}(15,t) = 0, \qquad t > 0$$

and the initial condition

$$u(x,0) = 25, \qquad 0 < x < 15$$