GD1685

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2000

MATHEMATICAL PHYSICS - MP150
FIRST ENGINEERING
Professor M.J. Sewell;
Professor J.N. Flavin;
Professor M. McCarthy;
Dr. P. O'Leary;
Dr. M.P. Tuite.
Time allowed: *Three* hours
Instructions: Full marks will be awarded for **SIX** complete questions

1. (a) Find the equation of the line through the points with coordinates $(-1, 0, 1)$ and $(1, -1, 3)$. Verify that the line contains the point $(-3, 1, -1)$.

   (b) Find the equation of the plane through the points $(0, 0, 0)$, $(2, 0, 1)$ and $(2, -1, 0)$.

   (c) The line of action of $\mathbf{F} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$ passes through the point $(1, 3, 1)$.

      i. Find the moment of $\mathbf{F}$ about the origin.

      ii. Find the moment of $\mathbf{F}$ about the point A at $(1, -1, 4)$.

      iii. Find the moment of $\mathbf{F}$ about the line L through A in the direction of $-\mathbf{i}$.

2. A car travels a distance of 300 m in a straight line at an average speed of 4m/s, going from rest with constant acceleration a_1 for 10 s, then moving with constant speed and then coming to rest with a constant retardation a_2 for the last 20 s of the motion. Show that $a_1 = .5\text{m/s}^2$and $a_2 = .25\text{m/s}^2$.

3. Two particles A and B have velocities $(2\mathbf{i} - 3\mathbf{j})$ m/s and $(v(\mathbf{i} + \mathbf{j}))$ m/s respectively. Find $\mathbf{r}_{B|A}$, the position of B relative to A for all t given that $\mathbf{r}_{B|A} = (-2\mathbf{i} - 4\mathbf{j})$ m at $t = 0$ and find the value of v such that A and B collide. If $v = 2$ m/s, find the time when A and B are closest together.

4. A particle, of mass 4kg, lying on a rough plane at 30^o, is attached by a light inextensible string over a smooth pulley at the top of the plane to another particle of mass 5kg hanging freely. The coefficient of friction between the 4kg mass and the plane is $1/2\sqrt{3}$.

   (a) Find the acceleration of the system when released from rest and the tension in the string.

(b) Find the final velocity and the distance moved after the masses have been moving for 3 secs.

5. A man A stands on a cliff of height h and notices a second man B on the seashore a distance h from the foot of the cliff. Simultaneously, A throws a stone away from the cliff with speed u and angle of projection α and B throws a stone with speed v and angle of projection β away from the cliff where both trajectories are in the same vertical plane. If the two stones collide show that

$$v \sin(\beta+\frac{\pi}{4}) = u \sin(\alpha + \frac{\pi}{4}).$$

6. A mass m is attached to a light rod of length 2.5 metres and undergoes simple harmonic pendulum motion. The mass is initially held at an angle of $\pi/6$ to the downward vertical and is released. (You may assume that $g \simeq 10\text{ms}^{-2}$.)

   (a) Describe the subsequent motion of the mass.

   (b) How long does it take for the mass to reach its lowest point and what is the velocity at that time?

7. A particle of mass 8 kg lies on a rough plane inclined at 60^o to the horizontal. The coefficient of friction between the mass and the plane is μ. The particle is attached by means of a light string passing over a smooth pulley to a freely hanging mass of 2 kg. The system is released from rest and the 8 kg mass moves down the plane. Find its speed after it has moved a distance d .

8. Two smooth elastic spheres A and B of equal radii and masses 10 kg and 9 kg respectively, lie on a smooth horizontal surface. A is projected towards B with a speed of 6 m/s and strikes B obliquely at an angle of 30 degrees with the line of centres. If $e = 1/2$, find the velocities of the two spheres after impact.

9. (a) A particle of mass 4 kg is attached by a light inextensible string of length 3 m to a fixed point. the particle moves in a horizontal circle at constant speed 2 m/s. Calculate the tension in the string.

   (b) A particle of mass 4 kg is attached to a hinged spring of stiffness 2 N, and natural length 1m. The particle is projected tangentially on a smooth horizontal surface with velocity 8m/s. Find the maximum distance reached.

10. Prove that the centroid of a uniform solid circular cone of height h is at a distance of $h/4$ from the centre of the base.

    From the base of such a cone a right circular portion of height $h_1 (< h/2)$, and with radius of base of the given cone is hollowed out. The top conical portion of height $h/2$ is then cut off. Find the distance between the centre of the base of the original cone and the centroid of the remaining solid.

11. A step ladder consists of two equal arms BA, AC freely hinged at A. The weight of AB is three times that of AC. Show that if the angle between BA and AC is steadily increased, then slipping first takes place at C.

    If BA, AC are placed so that $\prec BAC = 90$ degrees find the least value of μ to prevent slipping.