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NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2000

B Sc (Financial Mathematics and Economics) Year 1
MP 191 Mathematical Methods I
Professor M.J. Sewell;
Professor J.N. Flavin;
Dr.P.M. O'Leary.
Time allowed: *Two* hours
Full marks for TWO questions, ONE from each section

SECTION A

1. Consider the first order linear difference equation

$$x_{n+1} = Ax_n + B,$$

where A and B are constants. Show the the solution can be expressed in the form

$$\begin{aligned} x_n &= A^n x_0 + B(A^n - 1)/(A - 1), A \neq 1, \\ x_n &= x_0 + nB, A = 1. \end{aligned}$$

Use this formula to calculate the monthly repayments on a loan of £6000 to be paid back over 5 years at an interest rate of 6%, compounded annually.

2. Solve each of the following difference equations, and determine the behaviour of the solution sequence for large values of n.

(a)

$$x_{n+2} + 5x_{n+1} + 4x_n = 8, x_0 = 1, x_1 = 1,$$

(b)

$$x_{n+2} - x_{n+1} - 6x_n = n.$$

p.t.o.

SECTION B

1. The rent for an apartment in a college town varies with the distance of the apartment from the college.

   (a)Suppose that this relationship is given by

$$\frac{dy}{dx} = -(\frac{1}{x} + 3), 1 \leq x \leq 15,$$

   where y is the monthly rent and x is the distance. If $y = 250$ when $x = 1$, find y as a function of x. Find the value of y when $x = 5$.

   (b)If the relationship is of the form

$$\begin{aligned} \frac{dy}{dx} &= -.007y, \\ y(0) &= 300. \end{aligned}$$

   find the value of y when $x = 4$.

2. The function

$$P(t) = 1.535 + x(t)$$

   where $x(t)$ is the solution of the logistic equation

$$\frac{dx}{dt} = 0.023x(1 - \frac{x}{6.336}),\ x(0) = 0.767$$

   is suggested as a fit for the population of Sweden from 1800 to 1920. Here t is measured in years, $P(t)$ is in millions and $t = 0$ corresponds to 1800. Solve the equation, and compare the results that you get with the actual figures for the years 1930 (6.142), 1960 (7.495) and 1970 (8.04).