

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2000

Third Year Examination CS305

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Time allowed: *Two* hours

Full marks for TWO questions

1. Obtain the two-term Galerkin approximate solution to the problem defined by

$$-\frac{d^2u}{dx^2} + 9u = 1, 0 < x < 3$$

subject to the conditions $u(0) = 0, u(3) = 0$.

Use the trial functions $\phi_1 = x(3 - x), \phi_2 = x\phi_1$, or any other functions that you choose.

2. Let R denote the rectangular region $4 \geq x \geq 0, 2 \geq y \geq 0$. If $u(x, y)$ satisfies the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = y \text{ in } R$$

subject to $u = 0$ on the boundary, construct the Galerkin approximation to the problem using the trial function

$xy(4 - x)(2 - y)$.

3. Consider the problem

$$\begin{aligned}-\frac{d^2u}{dx^2} + 4u &= 1, 0 < x < 2 \\ u(0) &= u(1) = 0.\end{aligned}$$

The region $[0, 1]$ is divided into four regions of equal length, and the approximate solution is given by $\sum_{i=0}^4 c_i \phi_i(x)$, where $\phi_i(x)$ are linear functions satisfying the conditions $\phi_i(x_j) = \delta_{ij}$, where $\delta_{ij} = 1$ for $i = j$, $\delta_{ij} = 0$ for $i \neq j$, and where the nodes x_j denote the end-points of each element. In that case show that the above problem reduces to the set of equations

$$\begin{bmatrix} 26/3 & -20 & 0 \\ -20 & 26/3 & -20 \\ 0 & -20 & 26/3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$

4. Write a MAPLE procedure to solve *any* of the last three questions.
5. Give examples of the use of the following commands in MAPLE:
 - (i) `dsolve`;
 - (ii) `animate`;
 - (iii) `proc`.