OLLSCOIL NA hEIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND

International Postgraduate Hydrology Courses

M.Sc. Degree in Hydrology - Spring Examinations, 2000

HYDROLOGIC COMPUTATION

Examiners: Professor P. E. O'Connell
Professor C. Cunnane
Dr. M. Bruen

Time allowed is *three* hours. Attempt any *five* questions

1. Gaussian Elimination is a good method for solving general sets of simultaneous linear equations. However, in many practical applications in the water resources area the sets of equations to be solved are banded.

   (a) Give an example of a practical application in which the solution of a banded set of equations is required. Describe, briefly, how the equations are derived.

   (b) Explain how to take advantage of the banded structure to solve the equations more efficiently than the method of Gaussian Elimination.

2. (a) Derive the equation for the objective function used by the optimisation method when fitting the EV1 (Gumbel) distribution to an annual maximum data series, using the maximum likelihood method.

   (b) Explain, briefly, the subroutines and functions required to implement this solution on a computer and described how the relevant data is passed between them.

   Notes : The following formula, using standard notation, may be useful.

   Gumbel (EV1) distribution: $F(x) = \exp(-\exp(-\frac{x-u}{\alpha}))$

(contd.)

3. (a) Explain, briefly, Bellman's principle of optimality and specify the types of optimisation problem to which it applies.

(b) Solve the following, greatly simplified, optimisation problem using the method of Dynamic Programming.

A catchment authority has a reservoir which can supply water at a steady rate of 10 m^3/s. It must decide how much of this supply to allocate for three different uses (i) Public water supply, (ii) Industry and (iii) Irrigation. The net benefit it receives from each use is a nonlinear function of the allocation and is defined by the following Table. *Note that benefit can reduce if too much water is supplied.* The authority is required by law to provide at least 3 m^3/s for public supply and at least 1 m^3/s each to Industry and Irrigation. Find the allocations which provide the maximum benefit to the catchment authority.

Flow rate allocated to use (m^3/s)	Benefit (£/day) from allocation to Public supply	Benefit (£/day) from allocation to Industry	Benefit (£/day) from allocation to Irrigation
1	1000	700	900
2	1800	1300	1800
3	2400	1900	2600
4	2700	2400	2900
5	2800	2800	2900
6	2900	3200	2500
7	2900	3400	2500
8	2900	3500	2500
9	2900	3500	2000
10	2900	3600	2000

4. The following partial differential equations describe one-dimensional unsteady flow in a prismatic open channel with a rectangular cross-section. Derive the corresponding characteristic equations and explain how they might be solved.

Continuity : $$y\frac{\partial u}{\partial x} + u\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = 0$$

Momentum : $$\frac{\partial y}{\partial x} + \frac{u}{g}\frac{\partial u}{\partial x} + \frac{1}{g}\frac{\partial u}{\partial t} = S_0 - S_f$$

where

X : Distance along the channel

T : Time

Q : Discharge

A : Cross-sectional area

y : Depth of water

u : Average velocity of flow

S_0 : Slope of channel

S_f : Friction slope

(contd.)

5. The following explicit finite difference equation, in the usual notation, has been used for solving the partial differential equation for one-dimensional diffusion.

$$c_{i,j+1} = (1-2r)c_{i,j} + r(c_{i-1,j} + c_{i+1,j})$$

$$\text{where} \quad r = \frac{D\Delta t}{(\Delta x)^2}$$

The partial differential equation is $\quad \dfrac{\partial c}{\partial t} - D\dfrac{\delta^2 c}{\partial x^2} = 0$

(a) Show that if the finite difference scheme is stable the influence of the initial conditions decreases with time and the solution is determined by the boundary conditions

(b) Derive an expression for the truncation error in this formula and show that it goes to zero as Δx and Δt go to zero.

(c) Derive the particular relationship between Δx and Δt which reduces the truncation error from first order to second order.

6. The following simplified diagram shows a section through a long cofferdam in a porous river bed. Assemble the matrix of finite difference equations required to calculate the unknown piezometric heads at the grid points. Assume the bottom and both sides of the shaded area are impervious.

Note : You do not have to solve the equations