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THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

International Postgraduate Hydrology Course

M. Sc (Hydrology) - Summer Examinations, 2000

APPLIED HYDROLOGY I (EH 504)

Examiners: Professor P.E. O'Connell
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Time allowed is *three* hours. Attempt *any five* questions

(The Handout provided may be useful in one or more questions).

1. (a) In the context of **routing through** storage, distinguish *clearly* between

(i)	**concentrated storage**	and	**distributed storage,**
(ii)	**linear routing**	and	**non-linear routing,**
(iii)	**forward-routing**	and	**back-routing.**

(b) **Using the continuity equation,** or otherwise, show algebraically that, in the case of **reservoir routing,** for an isolated storm event, the peak of the outflow hydrograph lies at a point on the recession curve of the inflow hydrograph, this point corresponding to time t_m.

(c) In the context of the **Mandeville reservoir-back-routing procedure,**

(i) **outline the derivation of the equation**

$$I(t_1) = Q(t_1)\left[1 - \frac{\dfrac{dQ(t_1)}{dt}}{\dfrac{dQ_r(t_2)}{dt}}\right], \text{ for } t_1 \le T,\ t_2 \ge T$$

where T is the time to the point of inflection on the recession of the outflow hydrograph (i.e. the point at which the inflow is assumed to fall to zero) and $Q(t_1) = Q_r(t_2) \le Q(T)$, the subscript r denoting the outflow recession from storage only, i.e. for $t \ge T$

(ii) explain very *briefly* why, in the application of the **Mandeville equation,** it is necessary first to **model (i.e parameterize) the outflow recession curve** $Q_r(t)$ for $t \ge T$ and then extrapolate this curve back in time to the outflow peak time t_m.

(iii) indicate *very briefly* any other **difficulties associated with the practical application of the procedure.**

(d) Why is the **"back-routing"** process generally more sensitive to data errors than that of **"forward routing"**?

2. (a) In the context of **channel routing** for an isolated storm event, show that the outflow discharge peak value $Q_p = Q(t_p)$ is *higher* and occurs *later* than the discharge value associated with the time t_m when the inflow *just equals* the outflow.

(b) For the *special* case of the **Muskingum Channel Routing Model,** defined by the *linear* storage-inflow-outflow relation $S(t) = K\,[X.I(t) + (1-X).Q(t)]$ where K and X are constant parameters, show algebraically, *from first principles,* that, at time t_m ,

$$\frac{dQ(t_m)}{dt} = -\frac{X.\dfrac{dI(t_m)}{dt}}{(1-X)} > 0$$

so that $t_p > t_m$ and $Q_p = Q(t_p) > Q(t_m)$, as idicated in part (a) above.

(c) For the following discrete (Linear Transfer Function) form of **open channel routing** equation;

$$Q_m = C_o Q_{m-1} + C_1 I_m + C_2 I_{m-1}, \qquad \text{for } m = 0,1,2,....$$

either derive, from first principles, the classic **finite difference form** of the Muskingum model, yielding the coefficients as

$$C_o = \left[\frac{2K(1-X)-T}{2K(1-X)+T}\right], \quad C_1 = \left[\frac{T-2KX}{2K(1-X)+T}\right] \quad \text{and} \quad C_2 = \left[\frac{T+2KX}{2K(1-X)+T}\right]$$

or *outline* **the derivation of the Nash modification of the Muskingum model** (i.e. the discretely coincident form for linearly interpolated input) yielding the coefficients $C_o = e^{-\frac{T}{K}}$, $C_1 = \left[1-\frac{K}{T}(1-C_o)\right]$ and $C_2 = \left[\frac{K}{T}(1-C_o)-C_o\right]$.

(d) Show that, regardless of which set of coefficients are used in the routing equation for the **Muskingum model,**

(i) $(C_o + C_1 + C_2) = 1$, as the **model is conservative** and

(ii) the routing equation can also be expressed in the following **2-coefficient form;**

$$Q_m = Q_{m-1} + C_3(I_{m-1} - Q_{m-1}) + C_1(I_m - I_{m-1})$$

where $C_3 = (C_1 + C_2)$

(iii) the **convolution summation relation** $Q_m = \sum_{r=o}^{r=m} I_r h_{m-r}$

produces the same output/discharge series as the discrete routing equation of the **Muskingum model** provided that the system is initially relaxed and that the corresponding h_m series is defined as

$$h_o = C_1$$

$$h_1 = (C_o C_1 + C_2)$$

with $h_m = C_0^{m-1}(C_o C_1 + C_2) = C_o h_{m-1}$, for $m = 2,3,4,...$

(e) Which set of Muskingum routing coefficients would you consider to be the best? *Briefly* explain your answer!

3. (a) In the context of **unit hydrograph theory,** what (*very briefly*) do you understand by each of the following terms:

(i) base flow, **(ii) storm runoff,** **(iii) effective (excess) rainfall,**
(iv) the instantaneous unit hydrograph (the IUH), **(v) the unit S-curve, and**
(vi) the unit hydrograph of duration T (the TUH) .

(b) Briefly, why is it occasionally convenient or even necessary to **change the** ***duration*** **T of a unit hydrograph?**

(c) Suppose that the direct *storm runoff,* sampled at intervals of T = 2 hours, up to time t = 34 hours, due to 1 cm of *effective* rainfall falling *uniformly* on a catchment over a period of time 2T = 4 hours, is as tabulated below.

Time t (hours)	0	2	4	6	8	10	12	14	16
Storm Runoff (m^3s^{-1})	0.0	83.33	141.66	99.16	69.42	48.59	34.01	23.81	16.67
Time t (hours)	18	20	22	24	26	28	30	32	34
Storm Runoff (m^3s^{-1})	11.67	8.17	5.72	4.00	2.80	1.96	1.37	0.96	0.67

Estimate

(i) the value (in m^3s^{-1}) of the unit S-curve at time t = 8 hours

(ii) **the time to peak and the magnitude of the peak of the 4-hour unit** hydrograph h(2T, mT)

(iii) **the time to peak and the magnitude of the peak of the 6-hour unit** hydrograph h(3T, mT)

(iv) **the time to peak and the magnitude of the peak of the 2-hour unit** hydrograph h(T, mT).

(d) If the area of the catchment considered in (b) above is A = 400 km^2, **determine** the *theoretical* value of the **sum** of ***all*** of the **ordinates** of

(i) the **4-hour** unit hydrograph
(ii) the **6-hour** unit hydrograph
(iii) the **2-hour** unit hydrograph

where each unit hydrograph is sampled at intervals of T = 2 hours.

(e) Why (*in one sentence*) is it generally more difficult to derive a satisfactory estimate of a **shorter duration unit hydrograph from a longer one and vice** versa?

(f) What (*in one sentence*) do you perceive as the **fundamental weakness of the unit hydrograph theory ?**

(g) **If 1000 cumec hours of effective rainfall, falling** ***uniformly*** **in time and space over a period T=2 hours, on a catchment of area A=300 km^2 , produces a peak value of storm runoff of 120 cumecs, what peak value would be produced by 1.5 cm of effective rainfall,** also falling uniformly in time and space, over the same period over the same catchment?

4. (a) *List* the three main categories of discrete forms of continuous parametric linear models and state (with a *very brief* explanation) which forms can usefully be applied in rainfall-runoff modelling and in flood routing?

(b) Using the method of residues, or otherwise, show that the discrete unit impulse response series of a cascade of two unequal *discrete* linear reservoirs, of storage coefficients K_1 and K_2, defined by the linear difference equation

$$(1 + K_1\nabla)(1 + K_2\nabla)\,y_m = x_m$$

is given by

$$h_m = \left(\frac{1}{K_1 - K_2}\right)\left[\left(\frac{K_1}{1 + K_1}\right)^{m+1} - \left(\frac{K_2}{1 + K_2}\right)^{m+1}\right]$$

for $m = 0,1,2,3,..,$ ∇ is the backward difference operator and x_m and y_m are the input and output series respectively.

(c) To which of the categories referred to in part (a) does the model defined in part (b) belong?

(d) Demonstrate that

(i) the discrete linear model in part (b) above is "conservative"

(ii) the 1st moment of the h_m series about the discrete time origin is

$$_{h_m}M_1' = (K_1 + K_2)$$

(iii) the 2nd moment of the h_m series about its centroid is

$$_{h_m}M_2 = (K_1^2 + K_2^2) + (K_1 + K_2)$$

(e) If the input to a single *discrete* linear reservoir, (which is initially relaxed), consists of a simple geometric series of the form $x_m = \left(\frac{2}{3}\right)^{m+1}$, for $m = 0,1,2,....,$ and the corresponding output series is given by

$$y_m = \left[\left(\frac{4}{5}\right)^{m+1} - \left(\frac{2}{3}\right)^{m+1}\right], \quad \text{for } m = 0, 1, 2, ...$$

show that

(i) the storage constant of the discrete reservoir is $K = 4$ and

(ii) the discrete unit impulse response series of the reservoir has the form

$$h_m = \frac{1}{5}\left(\frac{4}{5}\right)^m, \qquad \text{for } m = 0, 1, 2, ...$$

(iii) the sum of that h_m series is unity, i.e. that $\sum_{m=0}^{\infty} h_m = 1,$

indicating that the discrete linear reservoir is also a conservative system.

5. (a) In the context of rainfall-runoff models, distinguish *very briefly but clearly* between

   (i) the empirical system-theoretic black-box models (lumped and semi-lumped)

   (ii) the conceptual catchment models (lumped and semi-lumped)

   (iii) the distributed (and semi-distributed) physically-based models

   and name *one example* of each category.

(b) Considering the Linear Perturbation Model (LPM) and the Variable Gain Factor Model (VGFM), indicate

   (i) to which model category each model belongs, and

   (ii) whether they are designed to address the phenomena of seasonality or of non-linearity (scale and/or shape) in the catchment response to rainfall, or both! (*Briefly* explain your answer!)

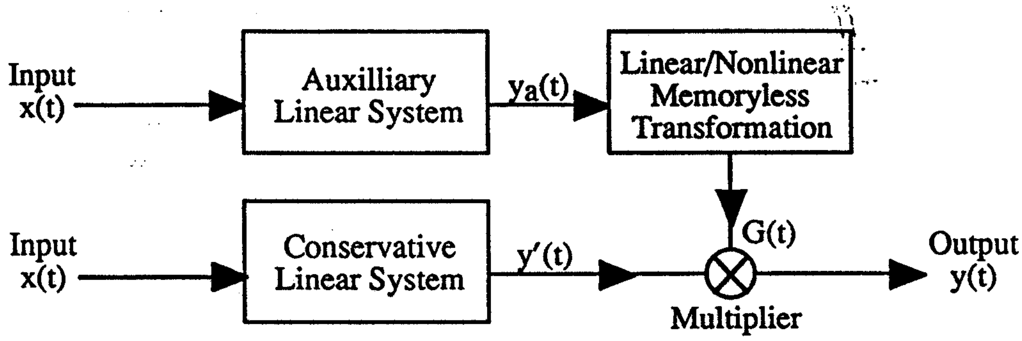
(c) Referring to the accompanying schematic representation of the general structure of the Variable Gain Factor Model (VGFM), explain *briefly but unambiguously* the form of the various components of the model, i.e. the relation between

   (i) $y_a(t)$ and $x(t)$,

   (ii) $y'(t)$ and $x(t)$,

   (iii) $G(t)$ and $y_a(t)$ and

   (iv) $y(t)$ and both $G(t)$ and $y'(t)$.



Schematic representation of the Variable Gain Factor Model (VGFM)

(d) Assuming a "linearly-varying" (LV) gain factor series of the form $G_i = a + bz_i$ in the VGFM, where z_i is the "wetness index" series and both parameter a and b are constants, *outline algebraically* how this form of the model can be calibrated *directly*, using the method of ordinary least squares, to provide a model output forecast equation of the form

$$\hat{y}_i = \sum_{j=1}^{m} \hat{W}''_j . x_{i-j+1} + \sum_{j=1}^{m} \hat{W}'_j .(z_i . x_{i-j+1})$$

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where $\hat{y}_i$ is the discharge forecast series, x_i is the rainfall series and $\hat{W}''_j$ and $\hat{W}'_j$ are the two identified weighting sequences, rather than the form

$$\hat{y}_i = G_i \sum_{j=1}^{m} w_j . x_{i-j+1}, \quad \text{with} \sum_{i=1}^{m} w_i = 1,$$

implied in the *theoretical* structure of the LVGFM form of the model.

(e) Suggest a modification of the structure of the VGFM, specifically in

(i) the form of the Auxiliary System described in your answer to part (c),

(ii) the form of linear variation of G_i with z_i, as specified in part (d),

which might lead to an improvement in the model efficiency?

6. (a) What are the two *principal* components or modules common to most conceptual rainfall-runoff models? *Very briefly*, explain the role of each of these modules in the overall operation of the model.

(b) With reference to the schematic line diagram of the standard (Liang) version of the Soil Moisture Accounting and Routing (SMAR) model, provided with the Formula Handout, explain *briefly but unambiguously* the operation of the model over a single data interval of time, (e.g. one day) when

(i) the rainfall depth R is *less than or equal to* the *model-estimated* potential evaporation depth E_p, (i.e. $R \leq E_p$), assuming the *all* the soil layers are full of moisture.

(ii) the rainfall depth R is *greater than* the *model-estimated* potential evaporation depth E_p, (i.e. $R > E_p$), assuming the *all* the soil layers are full of moisture.

(c) Suppose that the rainfall R and the pan evaporation E data, for three consecutive days, are given as follows :

Rainfall (mm/day)	0.0	15.0	40.0
Evaporation (mm/day)	30.0	20.0	10.0

The SMAR model was used in conjunction with the above synthetic data to simulate the three different components of *generated* runoff (i.e. that produced prior to routing), the corresponding evaporation depth for these three days and the final storage depths in the layers, as shown in the table overleaf, taking a total layer depth of Z=40 mm and a constant infiltration rate of Y = 100 mm/day. Find the values for the remaining three parameters (T, C and H) of the water-balance module of the SMAR model that were used to produce these results, assuming that both of the soil layers are initially full of moisture.

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Time Interval	r_1 (mm/day)	r_2 (mm/day)	r_3 (mm/day)	E_a (mm/day)	$S_{f,1}$ (mm)	$S_{f,2}$ (mm)
Day-1	0.0	0.0	0.0	26.6	0.0	13.4
Day-2	0.0	0.0	0.0	17.4	0.0	11.00
Day-3	2.13125	0.0	0.0	9.0	25.0	14.86875

Summary of the results of the SMAR model (Z=40mm, Y=100mm/day).

(Please note that r_1 denotes the direct generated runoff, r_2 the generated runoff in excess of infiltration (Y), r_3 the generated runoff in excess of the moisture capacity (Z) of the soil layers, $S_{f,1}$ and $S_{f,2}$ the final storage depths of layers one and two respectively at the end of each day, and E_a the actual evaporation).

(d) **Indicate very *briefly* any modification of the above version of the SMAR model that may substantially improve the efficiency of the model in simulating the outflows.**

7. (a) Distinguish clearly between a **'conceptual instantaneous unit hydrograph (IUH) model'** and **'a lumped conceptual catchment model'.**

(b) What are the **advantages,** if any, of the **conceptual IUH models** over the **non-parametric form of IUH model?**

(c) For the case of the **Nash** equal-reservoir cascade IUH/routing model, defined by the linear differential equation

$$(1+KD)^n y(t) = x(t)$$

demonstrate

(i) that the model is **conservative,**

(ii) that the **IUH function** of this model is the **Gamma density function**

$$\mathrm{IUH}(t) = h(o,t) = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-\left(\frac{t}{K}\right)}$$

(iii) **either** that the **r-th moment about the origin** of $h(o,t)$ has the form

$$_{h(t)}M_r' = \frac{(n+r-1)!}{(n-1)!}K^r, \quad \text{for } r = 0,1,2,3,,,,,$$

or that the **r-th cumulant** of $h(o,t)$ has the form

$$_{h(t)}K_r = (r-1)!\,nK^r, \quad \text{for } r = 1,2,3,4,.......$$

(iv) that the **dimensionless $m_3 : m_2$ relationship** for the IUH is given by

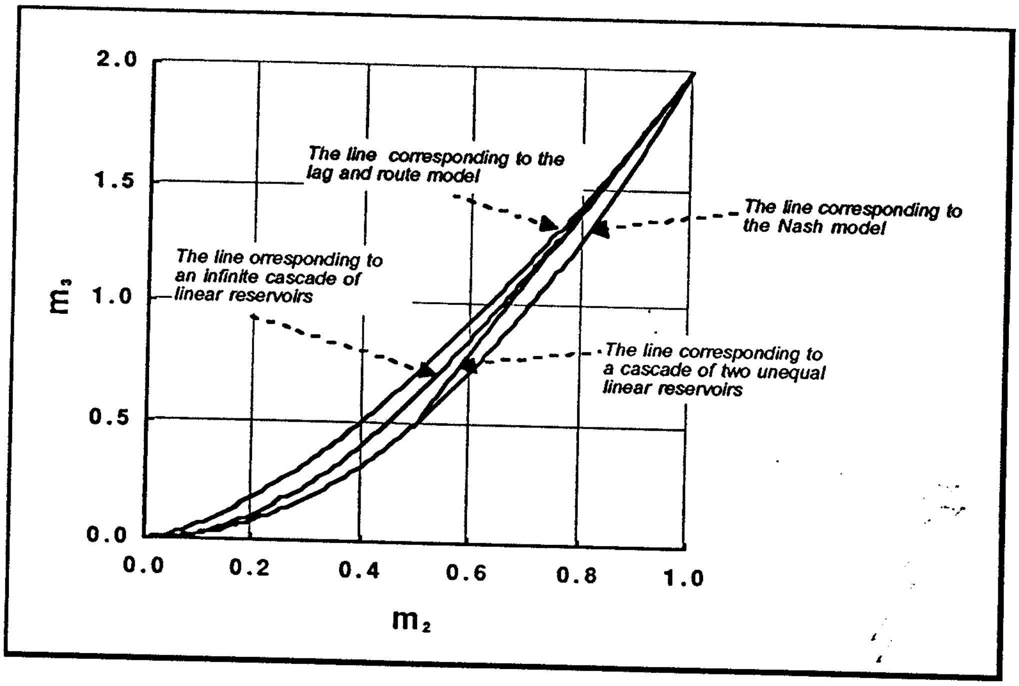
$$m_3 = 2m_2^2$$

Contd./

where $m_2 = \dfrac{{}_{h(t)}K_2}{\left({}_{h(t)}K_1\right)^2}$ and $m_3 = \dfrac{{}_{h(t)}K_3}{\left({}_{h(t)}K_1\right)^3}$

(d) With reference to the accompanying figure, which illustrates the $m_3 : m_2$ relation for a number of IUH models, what conclusion can be inferred concerning the generality of that Nash-cascade model?

(e) Explain briefly why, more that forty years after its introduction, the Nash-cascade model is still applied in the context of catchment modelling.



The m_3:m_2 diagram for different IUH models

8. (a) For a **linear reservoir**, defined by the storage-discharge relation $S(t) = K.Q(t)$, where K is a constant and t is time, show that its **discretely-coincident form** (CDF), for the case of **pulsed (i.e. histogram-type) inputs**, is

$$(1 - qB).Q_m = (1 - q).I_m \quad \text{for} \quad m = 0,1,2,3,......,$$

where $q = e^{-\frac{T}{K}}$, T is both the duration of the input pulses and the sampling interval, B is the backward shift operator and I_m and Q_m are the intensities of the input pulses and the sampled outflow hydrograph ordinates respectively.

(a) *Briefly* describe the **main steps** involved in the construction of the Clark **instantaneous unit hydrograph (IUH)**, with particular reference to the **determination of the isochrones of equal travel time, the time of concentration T_c, the time-area diagram and the storage coefficient** K of the continuous linear reservoir at the outlet of the catchment.
(A clear diagram would be helpful!)

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(b) The following table shows the **time-area diagram** (as areas between isochrones) for a catchment having an area $A = 110\ km^2$ and a time of concentration $T_c = 18$ hours. For the Clark model, find the IUH of this catchment. Assume that the storage coefficient of the linear reservoir at the outlet of watershed is $K = 12$ hours.

Travel Time *(hours)*	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Inter-isochrone area *(km^2)*	3	9	20	22	16	18	10	8	4

(c) What is the purpose or function of the **linear reservoir component** in the **Clark IUH model?**

(d) If, in practice, the time-area diagram of the catchment cannot be constructed in the usual way, what simple **parametric form or shape** has been used in the past to represent the **time-area diagram?**

(e) Does it really make sense to **separately parameterise the time area diagram,** rather than **directly parameterise the whole IUH** by, for example, the **Nash equal-reservoir cascade model?** *Briefly* explain your answer!

(f) In the context of a **digital elevation model** (DEM), a **GIS** and detailed information on the **distribution of rainfall** over the area of the catchment being available, does the **time-area diagram** still have a useful role to play in the modern practice of hydrology? *Briefly* explain your answer!