# OLLSCOIL NA hEIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

**International Postgraduate Hydrology Courses**

## Postgraduate Diploma in Hydrology - Spring Examinations, 2000

## HYDROLOGIC COMPUTATION

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Time allowed is *three* hours. Attempt any *five* questions

1. (a) Explain why pivoting ( swapping rows) may be necessary when solving sets of simultaneous linear equations with the method of Gaussian Elimination.

   (b) The ordinates of a unit hydrograph, h, can be estimated from effective rainfall, direct runoff data, using the method of least-squares, with the formula

$$X^T X h = X^T y$$

   If an effective rainfall series of (10, 30, 20) mm/hour produces a flood direct response equivalent to (1.5, 8.5, 17, 15, 7.5, 3.5, 1) mm/hour calculate the corresponding $X^TX$ matrix and $X^Ty$ vector for solving for the unit hydrograph ordinates.

2. (a) Explain the basis for the bisection (or secant) method for finding the root of an equation.

   (b) Show how to use the bisection (or secant) method to find the depth of critical flow in a trapezoidal channel of the following dimensions. *Only two iterations of the method are required.*

| | | | |
|---|---|---|---|
| Discharge | Q | : | 10 $m^3/s$ |
| Bottom width of channel | B | : | 2 m |
| Side slope of channel | z | : | 1:1 |

Note : $F_r^2 = \dfrac{Q^2 T}{g A^3}$

3. Derive an **implicit** finite difference formula for solving the partial differential equation representing one-dimensional diffusion.

$$\frac{\partial c}{\partial t} - D\frac{\partial^2 c}{\partial x^2} = 0$$

*(contd.)*

4. In a certain arid coastal region three sources of water are available for a new urban drinking water development, groundwater from wells, surface water from a nearby river and water from a desalination plant on the coast. Some of the chemical characteristics of these waters are listed in the table below, together with the maximum allowed concentration of each chemical in the drinking water.

| Chemical (unit) | Maximum Allowed | Source of water | | |
|---|---|---|---|---|
| | | Groundwater | River water | Desalinated water |
| Chloride ( mg/l) | 250 | 400 | 100 | 150 |
| Fluoride (mg/l) | 1 | 1.5 | 0.5 | 0.8 |
| Lead (mg/l) | 0.05 | 0.01 | 0.2 | 0 |
| Nitrate (mg/l) | 50 | 110 | 70 | 0 |

A mixture of water from the three sources is required which, for the least cost, gives a drinking water which does not require further treatment. The overall costs (calculated over the design life of the project) of supplying water from the three sources are 0.3 £/m$^3$, 0.6 £/m$^3$ and 1.5 £/m$^3$, for groundwater, river water and desalinated water respectively. Write down the equations which represent this problem and construct the starting Tableau for a linear programming solution. Do **one** step of the solution ( i.e. to the next tableau).

5. (a) Explain how to deal with impervious and constant head boundary conditions, when discretising Laplace's equation for steady groundwater flow, using the finite difference method.

Laplace's equation is: $$\frac{\delta^2 h}{\delta x^2}+\frac{\delta^2 h}{\delta y^2}=0$$

(b) Explain briefly how the complete set of finite difference equations can be solved.

(c) If the values of piezometric heads have been calculated for a finite difference grid, explain how to calculate the velocity of flow and the flow-rate through any cross-section.

6. The characteristic equations for the St. Venant equations representing one-dimensional unsteady flow in an open-channel are

$$\frac{dx}{dt} = v \pm c \qquad \textit{and} \qquad \frac{d(v \pm 2c)}{dt} = g(S_0 - S_f), \qquad \text{where}$$

| | | | | |
|---|---|---|---|---|
| X | distance along the channel (m) | v | velocity (m/s) | |
| T | time (s) | c | celerity (m/s) | $\approx \sqrt{gy}$ |
| $S_0$ | bed slope of channel | $S_f$ | friction slope | |

Let A and B be two points on a wide, rectangular, channel (i.e. hydraulic radius is close to the depth) , 1 kilometre apart and B downstream of A. The channel bed slope, $S_0$, is 0.0005 and Manning's n is 0.02. The velocity and depth at A and B at a reference time, t = 0, are

| | Point A | Point B |
|---|---|---|
| Velocity (m/s) | 1.5 | 1.2 |
| Depth (m) | 4 | 3 |

Assuming a wide channel ( i.e. hydraulic radius is equal to depth) estimate the velocity and depth at the point where the characteristic lines through A and B meet.