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THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

International Postgraduate Hydrology Course

Higher Diploma (Hydrology) - Summer Examinations, 2000

APPLIED HYDROLOGY I (EH 864)

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Time allowed is *three* hours. Attempt *any five* questions

(The Handout provided may be useful in one or more questions).

1. (a) In the context of a continuous linear time-invariant system, initially relaxed, demonstrate *graphically* the operation of the "convolution" of two time functions.

(b) Does the operation of **convolution** imply that the system is necessarily **conservative?** *(Briefly explain your answer!)*

(c) Is the application of the **convolution integral** generally a **practical option** in the hydrological context? *(Explain your answer in one sentence!)*

(d) For a system defined by the **convolution integral, derive relations** between:

(i) the **unit step response function** S(t) and the **unit impulse response function** h(o,t).

(ii) the **unit pulse** (of duration T) **response function** h(T,t) and the **unit impulse response function** h(o,t).

(iii) the **unit pulse response function** h(T,t) and the **unit step response** function S(t).

(e) Demonstrate that the **convolution summation relation**

$$y_m = \sum_{r=0}^{m} x_r h_{m-r} \qquad \text{for} \quad m = 0,1,2,3,.....,$$

is discretely coincident with the assumed convolution integral relation

$$y(t) = \int_0^t x(\tau).h(0,t-\tau).d\tau$$

between the *volumes* of the **effective rainfall blocks** and the *ordinates* of the **storm-runoff component** of the total discharge hydrograph.

(f) *State* clearly any **other conditions** which yield the **convolution summation** relation as a discretely coincident form of the **continuous linear model.**

2. (a) What *(very briefly!)* do you understand by each of the following terms:
flood routing, **reservoir routing,** **channel routing,**
linear routing. **forward-routing** and **back-routing.**

(b) In the context of **channel routing** for an isolated storm event, show that the outflow discharge peak value $Q_p = Q(t_p)$ is *higher* and occurs *later* than the discharge value associated with the time t_m when the inflow *just equals* the outflow.

(c) Demonstrate that the Muskingum channel routing model, defined by the **storage(S)-inflow(I)-outflow(Q)** relation $S(t) = K\,[X.I(t) + (1-X).Q(t)]$, where K and X are constant parameters, is a ***linear*** one?

(d) ***Outline* the derivation of the expressions for the coefficients C_0, C_1, and C_2 of the classic finite-difference form of the Muskingum channel routing model, having the general form**

$$Q_m = C_0 Q_{m-1} + C_1 I_m + C_2 I_{m-1}, \quad \text{for } m = 0,1,2, \ldots$$

and discuss *briefly* **the restrictions** (if any) **on the model parameters** K and X and on the data interval T.

(e) Show that, for the routing system to be **conservative,** the **coefficients** of any discrete form of the **Muskingum** model must satisfy the relation

$$(C_0 + C_1 + C_2) = 1$$

(f) Indicate *briefly* the **physically unrealistic characteristic** of the **unit impulse response function of the Muskingum model** and its **implication** in the application of the method.

3. (a) **Using the continuity equation,** or otherwise, show ***algebraically*** that, in the case of **reservoir routing,** for an isolated storm event, the **peak of the outflow hydrograph always lies at a point on the recession curve of the inflow hydrograph,** this point corresponding to time t_m.

(b) *Briefly* explain the **Modified Puls Method** for **routing** a ***given*** inflow hydrograph through a **non-linear reservoir,** given also the ***initial*** outflow and the storage-discharge relation for the outflow control.

(c) An artificial lake, of area 1 km², with vertical sides, is controlled by a broad-crested weir having the discharge/upstream-head relation $Q = 1.705\ B\ H^{3/2}$ m^3s^{-1}, the width of the weir being $B = 5$ m. Given that the initial water surface level in the lake is *just at* the crest of the weir, **check *any two* consecutive outflow values** in the following computer output table, for the ***given*** **inflow hydrograph, taking** the routing period $T = 6$ hours.

Time t (hours)	0	6	12	18	24	30	36	42
Inflow I (m^3s^{-1})	0	15	25	45	60	40	30	0
Outflow Q (m^3s^{-1})	0	0.5269	3.4350	10.5240	22.7720	31.9730	33.0474	26.7901

(d) **If the lake area A is reduced by 10%, e.g. by a landslide, what corresponding new value of the weir width B would yield *exactly* the same outflow data** as that given above, for the *same* inflow hydrograph and the *same* initial storage conditions?

4. (a) In the context of unit hydrograph theory, what (*very briefly*) do you understand by each of the following terms:

(i) base flow, (ii) storm runoff, (iii) effective (excess) rainfall,

(iv) the instantaneous unit hydrograph (the IUH), (v) the unit S-curve, and

(vi) the unit hydrograph of duration T (the TUH).

(b) Suppose that the direct *storm runoff*, sampled at intervals of T = 2 hours, up to time t = 34 hours, due to 1 cm of *effective* rainfall falling *uniformly* on a catchment over a period of time 2T = 4 hours, is as tabulated below.

Time t (hours)	0	2	4	6	8	10	12	14	16
Storm Runoff (m^3s^{-1})	0.0	83.33	141.66	99.16	69.42	48.59	34.01	23.81	16.67
Time t (hours)	18	20	22	24	26	28	30	32	34
Storm Runoff (m^3s^{-1})	11.67	8.17	5.72	4.00	2.80	1.96	1.37	0.96	0.67

Estimate

(i) the **value** (in m^3s^{-1}) of the **unit S-curve** at time t = 8 hours

(ii) the **time to peak** and the **magnitude of the peak** of the **4-hour unit** hydrograph h(2T, mT)

(iii) the **time to peak** and the **magnitude of the peak** of the **6-hour unit** hydrograph h(3T, mT)

(iv) the **time to peak** and the **magnitude of the peak** of the **2-hour unit** hydrograph h(T, mT).

(c) If the area of the catchment considered in (b) above is A = 400 km^2, **determine** the *theoretical* value of the **sum of *all* of the ordinates of**

(i) the **4-hour** unit hydrograph

(ii) the **6-hour** unit hydrograph

(iii) the **2-hour** unit hydrograph

where each unit hydrograph is sampled at intervals of T = 2 hours.

(d) Why is it generally more difficult to derive a satisfactory estimate of a shorter duration unit hydrograph from a longer one and vice versa?

(e) What do you perceive as the **fundamental weaknesses of the unit hydrograph theory** ?

5. (a) Explain, *very briefly*, why the **Linear Transfer (LTF) model,** defined by

$$(1+\phi_1 B+\phi_2 B^2+...+\phi_n B^n)y_m = (\theta_0+\theta_1 B+\theta_2 B^2+...+\theta_k B^k)x_m, \quad \text{for} \quad m = 0,1,2,3,.....,$$

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where B is the *backward shift* operator and $n > k$, is still widely used in deterministic hydrological modeling, particularly in the context of real-time river flow forecasting.

(b) Show that the Gain Factor (G) of the LTF model is given by

$$G = \frac{\theta_0 + \theta_1 + \theta_2 + \ldots + \theta_k}{1 + \phi_1 + \phi_2 + \ldots + \phi_n}$$

(c) For the discrete cascade model of two unequal discrete linear reservoirs, defined by

$$(1 + K_1\nabla)(1 + K_2\nabla)y_m = x_m \;, \quad \text{for } m = 0,1,2,3,\ldots.$$

where ∇ is the *backward difference* operator, with $\nabla = (1\text{-}B)$, show that

(i) the model is conservative,

(ii) it's unit impulse response series is given by

$$h_m = \frac{(1-q_1)(1-q_2)}{(q_1 - q_2)}\left[q_1^{m+1} - q_2^{m+1}\right], \quad \text{for } m = 0,1,2,3,\ldots\ldots,$$

where $q_1 = \dfrac{K_1}{1+K_1}$ and $q_2 = \dfrac{K_2}{1+K_2}$,

(iii) its lag (i.e. the 1st moment about the origin) is $M_1' = (K_1 + K_2)$.

(d) If only two isolated blocks of effective rainfall, 2cm and 1cm in depth, each of duration $T = 2$ hours, fall on a catchment of area $A = 360$ km^2, determine the first five ordinates of the storm runoff, at 2-hour intervals, assuming that the form of LTF model in part (c) above is appropriate, with $K_1 = 4$ and $K_2 = 2$, using *either* the linear difference equation *or* convolution summation.

6. (a) For a linear reservoir, defined by the storage-discharge relation $S(t) = K.Q(t)$, where K is a constant and t is time, show in outline that its discretely-coincident form (CDF), for the case of pulsed (i.e. histogram-type) inputs, is

$$(1 - qB).Q_m = (1 - q).I_m \qquad \text{for } m = 0,1,2,3,\ldots\ldots,$$

where $q = e^{-\frac{T}{K}}$, T is both the duration of the input pulses and the sampling interval, B is the backward shift operator and I_m and Q_m are the intensities of the input pulses and the sampled outflow hydrograph ordinates respectively.

(b) Describe the main steps involved in the construction of the Clark instantaneous unit hydrograph (IUH), with particular reference to the determination of the isochrones of equal travel time, the time of concentration T_c, the time-area diagram and the storage coefficient K of the continuous linear reservoir at the outlet of the catchment.
(A clear diagram would be helpful!)

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(c) The following table shows the **time-area diagram** (as areas between isochrones) for a catchment having an area $A = 110\ km^2$ and a time of concentration $T_c = 18$ hours. For the **Clark model, find the IUH** of this catchment. Assume that the storage coefficient of the linear reservoir at the outlet of watershed is $K = 12$ hours.

Travel Time (hours)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Inter-isochrone area (km^2)	3	9	20	22	16	18	10	8	4

(d) Is the Clark IUH model still used ? *Briefly* explain your answer!

7. (a) What are the two *principal* components or modules common to most **conceptual rainfall-runoff models?** ***Very briefly,*** **explain the role of each of these modules in the overall operation of the model.**

(b) With reference to the schematic line diagram of the **Liang version** of the **Soil Moisture Accounting and Routing (SMAR) model,** provided with the **Formula Handout,** explain *briefly but unambiguously* the operation of the model over a single data interval of time, (i.e. one day) when

(i) when the rainfall depth is *less than or equal to* the potential evapotranspiration depth, assuming that *all* of the soil layers are full of moisture.

(ii) when the rainfall depth is *greater than* the potential evapotranspiration depth assuming that *not all* of the soil layers are full of moisture.

(c) Suppose that the **optimised parameters of the water balance component** of the SMAR model are given as:

$T = 0.9$, $H = 0.25$, $Y = 100$ mm/day, $Z = 40$ mm and $C = 0.8$ and

and that the **rainfall (R)** and **pan-evaporation (E)** data for three **consecutive days** on the chosen catchment are as follows:

Rainfall in mm/day:	0	15	40
Pan Evap. in mm/day:	30	20	10

Assuming that *all* of the soil layers are *initially* full of moisture, **check the results** in the table below for the **outputs of the water balance module.**

Time Interval	r_1 (mm/day)	r_2 (mm/day)	r_3 (mm/day)	E_a (mm/day)	$S_{f,1}$ (mm)	$S_{f,2}$ (mm)
Day-1	0.0	0.0	0.0	26.6	0.0	13.4
Day-2	0.0	0.0	0.0	17.4	0.0	11.00
Day-3	2.13125	0.0	0.0	9.0	25.0	14.86875

Summary of results for the water balance module of the SMAR model

(Please note that r_1 denotes the direct generated runoff, r_2 the generated runoff in excess of infiltration (Y), r_3 the generated runoff in excess of the moisture capacity (Z) of the soil layers, $S_{f,1}$ and $S_{f,2}$ the final storage depths of layers one and two respectively at the end of each day and E_a the actual evaporation).